Received: *August 7, 2001*, Accepted: *September 19, 2001*

Towards massless higher spin extension of $D = 5$, $N = 8$ gauged supergravity

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Abstract: The AdS_5 superalgebra PSU$(2,2|4)$ has an infinite dimensional extension, which we denote by $hs(2,2|4)$. We show that the gauging of $hs(2,2|4)$ gives rise to a spectrum of physical massless fields which coincides with the symmetric tensor product of two AdS_5 spin-1 doubletons (i.e. the $N = 4$ SYM multiplets living on the boundary of AdS_5). This product decomposes into levels $\ell = 0, 1, 2, \ldots, \infty$ of massless supermultiplets of PSU$(2,2|4)$. In particular, the $D = 5, N = 8$ supergravity multiplet arises at level $\ell = 0$. In addition to a master gauge field, we construct a master scalar field containing the $s = 0, 1/2$ fields, the anti-symmetric tensor field of the gauged supergravity and its higher spin analogs. We define the linearized constraints and obtain the linearized field equations of the full spectrum, including those of $D = 5, N = 8$ gauged supergravity and in particular the self-duality equations for the 2-form potentials of the gauged supergravity (forming a 6-plet of SU(4)), and their higher spin cousins with $s = 2, 3, \ldots, \infty$.

Keywords: Field Theories in Higher Dimensions, AdS-CFT Correspondance, Space-Time Symmetries, Supergravity Models.

JHEP09(2001)025

Contents

Higher spin gauge theories, for sometime considered in their own right (see, for example, [1] for a review), are likely to find their niche in M-theory, where it is natural to expect similar structures in the limit of high energies. In fact, the first indication of a possible connection between higher spin gauge theory and the physics of extended objects was pointed out long ago [2] in the context of the eleven dimensional supermembrane on $AdS_4 \times S^7$. More recently, tensionless type IIB closed string theory in a background with non-zero three-brane charge has been argued to be described by a higher spin gauge theory expanded around AdS_5 [3, 4]. The tensionless limit requires vanishing string coupling. The higher spin gauge theory therefore describes stringy interactions of a new kind. In the limit of large three-brane charge, i.e. weak curvature, the five-dimensional Planck scale is much larger than the inverse AdS-radius, and the higher spin gauge theory has an effective field theory description in terms of a curvature expansion valid at energies much smaller than the Planck scale.

Anti-de Sitter spacetime arises naturally in higher spin gauge theory. This suggests that higher spin gauge theory may play a role in understanding the strong version of the Maldacena conjecture, i.e. not relying on taking the low-energy limit. In this context, tensionless closed strings would be dual to tensionless open strings with vanishing t' Hooft coupling. A starting point is to examine currents formed out of free superdoubletons in 4d Minkowski space.

The superdoubleton representations in question are the ultra short representations of AdS_5 superalgebra PSU$(2,2|4)$ which have a fixed radial dependence in AdS_5 and thus they live on the boundary of AdS_5 [6]. A group theoretically precise definition will be given later but for the present discussion let us note that these are massless representations of the $4d$ super Poincaré algebra which admit the realization of the $4d$ superconformal symmetry, which is isomorphic to the AdS_5 supersymmetry. Moreover, there are infinitely many such AdS_5 doubleton supermultiplets which are listed in Tables 1 and 2. Note that the shortest one is the (self-conjugate) Yang-Mills supermultiplet, and that the usual $4d, N=4$ supergravity multiplet also figures in the list (the level one supermultiplet in table 1). In fact, the tensor product of any pair of superdoubletons decomposes into an infinite set of AdS_5 massless supermultiplets. We will focus our attention on the higher spin gauge theory based on the $N=4SYM$ doubletons but we shall come back to this point in the the final section.

Assuming the strong version of the Maldacena conjecture, the boundary currents mentioned above should be formed strictly out of the Yang-Mills multiplet (with global SU(N) symmetry). The symmetric tensor product of two such multiplets decomposes into an infinite set of massless supermultiplets of the AdS_5 superalgebra [6, 7, 8] which arrange themselves into levels $\ell=0,1,2,\ldots,\infty$ which contain the $D=5, N=8$ gauged supergravity multiplet at level $\ell=0$. This suggests that all the conserved boundary currents made out of two AdS_5 Yang-Mills doubletons (in the $g_{YM}\to 0$ limit) couple naturally to the massless higher spins of the higher spin $D=5, N=8$ supergravity theory considered here. This in itself, of course, is not enough of an evidence for a connection with string theory. See, however, [3, 4], where further arguments are given in favor of such a connection.

In trying to make contact with string theory, an important issue is whether open string interactions survive the tensionless limit, and in that case, whether these can be accommodated in the superdoubleton theory, perhaps in the form of higher derivative deformations. These interactions would be different in nature from the ordinary Yang-Mills interactions, which preserve conformal invariance but break higher spin symmetries [9].

Motivated by these considerations, we recently constructed a bosonic higher spin algebra extension of the AdS_5 group based on spin zero doubletons, and gave its linearized gauge theory in five dimensions [1]. In this paper we generalize this result to an extension of the the AdS_5 superalgebra PSU$(2,2|4)$, which we shall denote by

$hs(2,2|4)$, based on the Yang-Mills superdoubleton. The main results of this paper are:

- The identification of the symmetry group of the higher spin gauge theory as the higher spin extension of the AdS_5 superalgebra $\mathrm{PSU}(2,2|4)$ in which an ideal generated by a central element is modded out [10].

- The definition of the massless spectrum as the product of two AdS_5 Yang-Mills superdoubletons.

- The identification of the spectrum with the physical field content of one-form and zero-form master fields subject to (linearized) constraints.

- The linearized field equations of the full spectrum, including those of gauged $D = 5, N = 8$ supergravity [11, 12] and in particular the self-duality equations for the 2-form potentials of the gauged supergravity (forming a 6-plet of $\mathrm{SU}(4)$), and their higher spin cousins with $s = 2, 3, \ldots, \infty$.

The analysis in this paper is linearized. However, the field content is complete in the sense that the full spectrum of the higher spin algebra is included, as required by unitarity and ultimately by the consistency of the full interacting theory. In a recent paper [13], certain cubic interactions of a bosonic higher spin theory in five dimensions were constructed using an action. In particular, these interactions do not involve the matter fields which are essential for the description of the full interacting theory. It is clearly desirable to construct the full interactions of the $D = 5, N = 8$ higher spin gauge theory, which we believe should exist. We are currently studying this problem.

The paper is organized as follows. In section 1, we give the necessary details of the representation theory of $\mathrm{SU}(2,2|4)$, with particular emphasis on the oscillator realization. In section 2, the higher spin extension $hs(2,2|4)$ of $\mathrm{PSU}(2,2|4)$ is defined. In section 3, the spectrum of states as the symmetric product of two $N = 4$ SYM doubletons is defined (the decomposition of the spectrum under $\mathrm{PSU}(2,2|4)\times\mathrm{U}(1)_Y$ is shown in appendix A). In section 4, the kinematics of the field theoretical realization of $hs(2,2|4)$ as a gauge theory in five-dimensional spacetime is described. In section 5, the field content of the master scalar field is determined. In section 6, the linearized constraints and the proof of their integrability are given. In section 7, the resulting field equations, including self-duality equations for higher spin fields generalizing those of the $D = 5$, $N = 8$ gauged supergravity theory, are found (the details of harmonic analysis used to compute lowest energies are provided in appendix B). In section 8, we comment further on our results and suggest future directions.

1. Doubletons and massless irreps of PSU$(2,2|4)$

The generators of SU$(2,2|4)$ (and also its higher spin extension) can be constructed in terms of the bosonic oscillators y_α ($\alpha = 1,\dots,4$), which are SO$(4,1)$ Dirac spinors, and the fermionic oscillators θ_i ($i = 1,\dots,4$), which are in the fundamental representation of SU(4). The oscillator algebra reads

$$y_\alpha \star \bar{y}^\beta - \bar{y}^\beta \star y_\alpha = 2\delta_\alpha^\beta\,, \qquad \bar{\theta}^i \star \theta_j + \theta_j \star \bar{\theta}^i = 2\delta_j^i\,, \tag{1.1}$$

where $\star$ denotes the operator product. The remaining (anti)commutators vanish. We also define a Weyl ordered product as follows:

$$\begin{aligned} y_\alpha \bar{y}^\beta &= y_\alpha \star \bar{y}^\beta - \delta_\alpha^\beta\,, & \bar{y}^\alpha y_\beta &= \bar{y}^\alpha \star y_\beta + \delta_\beta^\alpha\,, \\ \bar{\theta}^i \theta_j &= \bar{\theta}^i \star \theta_j - \delta_j^i\,, & \theta_i \bar{\theta}^j &= \theta_i \star \bar{\theta}^j - \delta_i^j\,. \end{aligned} \tag{1.2}$$

The Weyl ordered product extends in a straightforward fashion to arbitrary polynomials of the oscillators; see [1] for details and the SO$(4,1)$ spinor conventions. The generators of SU$(2,2|4)$ consist of the bosonic SU$(2,2) \times$ SU$(4) \times$ U$(1)_Z$ generators denoted by $M^\alpha{}_\beta$, $T^i{}_j$, Z, respectively, and the supersymmetry generators Q^i_α. They can be realized as

$$\begin{aligned} M^\alpha{}_\beta &= \frac{1}{2}\bar{y}^\alpha y_\beta - \frac{1}{4}\delta^a_b K\,, \\ T^i{}_j &= \frac{1}{2}\bar{\theta}^\alpha \theta_j - \frac{1}{4}\delta^i_j X\,, \\ Z &= \frac{1}{2}(K + X)\,, \\ Q^i_\alpha &= \bar{\theta}^i y_\alpha\,, \end{aligned} \tag{1.3}$$

where

$$K \equiv \frac{1}{2}\bar{y}y\,, \qquad X \equiv \frac{1}{2}\bar{\theta}\theta\,. \tag{1.4}$$

In tensorial basis the SU$(2,2)$ generators are $M_{AB} = (\Sigma_{AB})_\alpha{}^\beta M^\alpha{}_\beta$ ($A = 0,1,2,3,5,6$) where Σ_{AB} are the van der Waerden symbols of SO$(4,2)$. The AdS energy generator is $E = M_{60}$. The generator Z commutes with all the generators and therefore acts like a central charge. By factoring out this Abelian ideal one obtains the simple Lie superalgebra PSU$(2,2|4)$. As we shall see later, the higher spin algebra we will work with is a natural extension of PSU$(2,2|4)$. There also exists an outer automorphism group U$(1)_Y$ generated by X. The U$(1)_Y$ charge is denoted by Y. In summary, the PSU$(2,2|4) \times$ U$(1)_Z \times$ U$(1)_Y$ algebra takes the schematic form $[\Lambda, \Lambda] = \Lambda + Z$, $[Z, \Lambda] = 0$ and $[Y, \Lambda] = \Lambda$, where Λ are the generators of PSU$(2,2|4)$ and Z appears only in the anti-commutator of two conjugate supersymmetry generators. With Z modded out, the PSU$(2,2|4)$ algebra closes and Y acts as an outer automorphism. In fact, this structure will generalize to the higher spin algebra.

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$\lvert Z\rvert \backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\cdots$
0	6	4	1					
1	1	4	6	4	1			
2			1	4	6	4	1	
3					1	4	6	$\cdots$
4							1	$\cdots$
$\vdots$								

Table 1: The superdoubletons with integer Z. The spin is defined as $s = j_L + j_R$ and the entries denote the SU(4) representations. Suitable reality conditions need to be imposed. The $\mathrm{U}(1)_Y$ charges of 1, 4 and 6 are 0, ± 1 and ± 2, respectively. For $|Z| \geq 1$ each supermultiplet consists of a superdoubleton and its CPT-conjugate (with opposite signs of Z) combined together. The $Z = 0$ multiplet is the (self-conjugate) $d = 4, N = 4$ Yang-Mills supermultiplet.

$2\lvert Z\rvert \backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	$\cdots$
1	4	6 + 1	4	1					
3		1	4	6	4	1			
5				1	4	6	4	1	
7						1	4	6	$\cdots$
9								1	$\cdots$
$\vdots$									

Table 2: The superdoubletons with half-integer $|Z|$; see caption of table 1. Here suitable reality as well as self-duality conditions need to be imposed.

Representations of $\mathrm{PSU}(2,2|4)$ are representations of $\mathrm{SU}(2,2|4)$ with vanishing central charge Z. Physical representations of $\mathrm{SU}(2,2|4)$ consist of a multiplet of lowest weight representations $[D(j_L, j_R; E_0) \otimes R \otimes Z]_Y$ of the bosonic subalgebra $B = \mathrm{SU}(2,2) \times \mathrm{SU}(4) \times \mathrm{U}(1)_Z$, which are characterized by ground states labeled by the energy E_0, the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \subset \mathrm{SU}(2,2)$ spins (j_L, j_R), an SU(4) irrep R, a central charge Z and an $\mathrm{U}(1)_Y$ charge Y [6]. A subset of them, forming the Clifford vacuum, is also the ground state of $\mathrm{SU}(2,2|4)$. The requirement that it is also annihilated by fermionic energy lowering operators puts restriction on its labels. Action by the fermionic energy raising operators then generate a supermultiplet.

The oscillator realization gives rise to unitary supermultiplets; see appendix A. The Fock space of a single set of oscillators decomposes into the doubleton supermultiplets listed in table 1 and table 2. These have either $E_0 = j_L + 1$, $j_R = 0$ or $E_0 = j_R + 1$, $j_L = 0$. The central charge is given by $Z = j_L - j_R + \frac{1}{2}Y$. Their $\mathrm{SU}(4) \times \mathrm{U}(1)_Y$ content is given in the tables. The decomposition of the oscillator

$\ell\backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	42	48	27	8	1									
1	1	8	28	56	70	56	28	8	1					
2					1	8	28	56	70	56	28	8	1	
3									1	8	28	56	70	$\cdots$
4													1	$\cdots$
$\vdots$														

Table 3: The symmetric tensor product of two $N = 4$ SYM doubletons arranged into levels $\ell = 0, 1, 2, \ldots$ of $N = 8$ AdS_5 superalgebra multiplets. Each level appears in the product once and only once, and consists of $USp(8)$ representations, some of which are reducible: $28 = 27 + 1$, $56 = 48 + 8$, $70 = 42 + 27 + 1$. Under $\mathrm{SU}(4) \times \mathrm{U}(1)_Y$: $8 = 4_1 + \bar{4}_{-1}$, $27 = 15_0 + 6_2 + \bar{6}_{-2}$, $42 = 20'_0 + 10_2 + \overline{10}_{-2} + 1_4 + \bar{1}_{-4}$, $48 = 20_1 + \overline{20}_{-1} + 4_3 + \bar{4}_{-3}$. The $\mathrm{U}(1)_Y$ charges are determined by $j_L - j_R + \frac{1}{2}Y = 0$. The level 0 multiplet is the usual $D = 5, N = 8$ supergravity multiplet. The states in the $s < 1$ sector arise as the physical states in the master scalar field Φ, as shown in table 6. For $s \geq 1$, the states with $Y = 0, \pm 1$ arise as physical states in the gauge fields corresponding to the generators of the higher spin algebra listed in table 4. Those with $Y = \pm 2, \pm 3, \pm 4$ arise as physical states in the master scalar field Φ. With the exception noted in table 6, these have dual gauge fields corresponding to the generators listed in table 5.

Fock space into superdoubletons is such that each allowed value of Z (integer and half-integer) occurs once and only once.

By considering tensor products of oscillator Fock spaces, one can construct massless and massive supermultiplets. The two-fold tensor product gives rise to massless representations [6]. In particular, the product of a superdoubleton and its CPT-conjugate gives rise to vanishing central charge and energies obeying[1]

$$E_0 = \begin{cases} j_L + j_R + 2 \text{ for } j_L + j_R \geq 1 \\ 2 + \dfrac{|Y|}{2} \qquad \text{for } j_L + j_R = 0\,, \dfrac{1}{2} \end{cases} \tag{1.5}$$

The $\mathrm{U}(1)_Y$ charges are determined by $j_L - j_R + \frac{1}{2}Y = 0$. In particular, the symmetric product of two Yang-Mills supermultiplets, which are CPT self-conjugate superdoubletons with $Z = 0$ and play an important role in the construction of our higher spin theory, decomposes into massless supermultiplets as shown in table 3.

2. The higher spin superalgebra $hs(2, 2|4)$

The definition of the higher spin extension $hs(2, 2)$ of $\mathrm{SU}(2, 2)$, with spectrum given by the product of two (bosonic) scalar doubletons, was first given in [1]. An important

[1]More generally, there are 'novel' massless multiplets [7] that have energy $E_0 > j_L + j_R + 2$.

feature is the modding out of an abelian ideal, which is generated by K in the bosonic case. In this section we define a higher spin extension $hs(2,2|4)$ of $\mathrm{PSU}(2,2|4)$ (denoted by $ho_0(8,8|8)$ in [10]) by the coset $\mathcal{G}/\mathcal{I}$, where $\mathcal{G}$ is a Lie supersubalgebra of the algebra $\mathcal{A}$ of arbitrary polynomials of the oscillators in (1.1) and $\mathcal{I}$ is an ideal of $\mathcal{G}$ generated by the central element Z.

We first define a set of linear maps τ_η, labeled by a unimodular complex parameter η, acting on a Weyl ordered function F of oscillators as follows ($|\eta|=1$):

$$\tau_\eta(F(y_\alpha,\bar{y}^\alpha,\theta_i,\bar{\theta}^i)) = F(\eta y_\alpha,-\bar{\eta}\bar{y}^\alpha,\eta\theta_i,-\bar{\eta}\bar{\theta}^i)\,. \tag{2.1}$$

These maps act as anti-involutions of $\mathcal{A}$:

$$\tau_\eta(F\star G) = (-1)^{FG}\tau_\eta(G)\star\tau_\eta(F)\,. \tag{2.2}$$

The Lie superalgebra $\mathcal{G}$ is defined to be the subspace of $\mathcal{A}$ consisting of Grassmann even elements P obeying ($|\eta|=1$)[2]

$$\mathcal{G}\ :\quad \tau_\eta(P) = -P\,,\qquad (P)^\dagger = -P\,, \tag{2.3}$$

and with Lie bracket

$$[P,Q] = P\star Q - Q\star P\,. \tag{2.4}$$

We decompose $\mathcal{G}$ into levels labeled by $\ell = 0,1,2,\ldots$, such that the ℓth level is given by all elements of the form:[3]

$$\begin{aligned} P^{(k)}(m,n;p,q) &= \frac{1}{m!n!p!q!}Z^{*k}\star \\ &\quad\star\left[P^{(k)\beta_1\ldots\beta_n\ j_1\ldots j_q}_{\ \ \alpha_1\ldots\alpha_m\,i_1\ldots i_p}\bar{y}^{\alpha_1}\cdots\bar{y}^{\alpha_m}y_{\beta_1}\cdots y_{\beta_n}\bar{\theta}^{i_1}\cdots\bar{\theta}^{i_p}\theta_{j_1}\cdots\theta_{j_q}\right], \qquad (2.5)\\ m+n+p+q &= 4\ell+2\,,\qquad m+p=n+q\,,\qquad n+q = 1 \bmod 2\,,\\ P^{(k)\gamma\beta_1\ldots\beta_{n-1}\ j_1\ldots j_q}_{\ \ \gamma\alpha_1\ldots\alpha_{m-1}\,i_1\ldots i_p} &= 0\,, \qquad (2.6)\end{aligned}$$

where we use the notation

$$Z^{\star k} = \underbrace{Z\star\cdots\star Z}_{k\text{ factors}}\,. \tag{2.7}$$

This basis yields the following decomposition of $\mathcal{G}$:

$$\mathcal{G} = \mathcal{G}^{(0)} + Z\star\mathcal{G}^{(1)} + Z^{\star 2}\star\mathcal{G}^{(2)} + \cdots\,. \tag{2.8}$$

[2]The algebra $\mathcal{G}$ can be enlarged by restricting η such that $\eta^M=1$ for some integer M. For example, taking $\eta = i$ one obtains a higher spin algebra, denoted by $ho(8,8|8)$ [10], with the finite subalgebra $OSp(8|8)\supset \mathrm{SO}(8)\times Sp(8,R)$, where $\mathrm{SO}(8)\supset\mathrm{SU}(4)$ and $Sp(8,R)\supset\mathrm{SU}(2,2)$.

[3]One can equivalently choose a basis in which the elements are traceless in the superindex $R=(\alpha,i)$. The two bases are related by redefinitions involving finite linear combinations.

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Since Z is central and $\tau_\eta(Z) = -Z$, it follows from (3) that $\tau(P^{(k)}) = (-1)^{1+k} P^{(k)}$. Hence $\mathcal{G}^{(k)}$ is isomorphic to $\mathcal{G}^{(0)}$ or $\mathcal{G}^{(1)}$ for k even or odd, respectively. Since Z is also hermitian, the traceless multi-spinors $P^{(k)}$ obey the reality condition:

$$\bar{P}^{(k)\beta_1\dots\beta_n\, j_1\dots j_q}{}_{\alpha_1\dots\alpha_m\, i_1\dots i_p} = -(-1)^{\frac{1}{2}p(p+1)+\frac{1}{2}q(q+1)} P^{(k)\beta_1\dots\beta_n\, j_1\dots j_q}{}_{\alpha_1\dots\alpha_m\, i_1\dots i_p}\,, \tag{2.9}$$

where the conjugation is defined as

$$\bar{F}^{\delta_1\dots\delta_n\, j_1\dots j_q}{}_{\gamma_1\dots\gamma_m\, i_1\dots i_p} = (i\Gamma^0)_{\gamma_1}{}^{\alpha_1}\cdots(i\Gamma^0)_{\gamma_m}{}^{\alpha_m}\left(F^{\alpha_1\dots\alpha_m\, i_1\dots i_p}{}_{\beta_1\dots\beta_n\, j_1\dots j_q}\right)^\dagger (i\Gamma^0)_{\beta_1}{}^{\delta_1}\cdots(i\Gamma^0)_{\beta_n}{}^{\delta_n}\,. \tag{2.10}$$

The hermitian conjugation interchanges the order of Grassmann odd quantities. The degeneracy in $\mathcal{G}$ due to Z having spin zero suggests that Z should be eliminated from the higher spin algebra. The Lie bracket (2.4) induces a set of brackets with the following structure:

$$[\cdot,\cdot]\ :\ \mathcal{G}^{(k_1)}\times\mathcal{G}^{(k_2)}\ \to\ \mathcal{G}^{(k_1+k_2)}+\mathcal{G}^{(k_1+k_2+1)}+\cdots\,, \tag{2.11}$$

where the direct sum is finite. This structure is due to the fact that the Lie bracket (2.4) does not preserve the tracelessness condition in (2.6). Thus the higher spin algebra cannot simply be the restriction of $\mathcal{G}$ to $\mathcal{G}^{(0)}$. Instead we let

$$\mathcal{I} = Z\star\mathcal{G}^{(1)} + Z^{\star 2}\star\mathcal{G}^{(2)}+\cdots\,. \tag{2.12}$$

This space forms an ideal in $\mathcal{G}$, i.e. $[\mathcal{G},\mathcal{I}] = \mathcal{I}$. We next observe that $\mathrm{U}(1)_Y$ acts as an outer automorphism of $\mathcal{G}$, i.e. $\mathcal{G}\setminus\mathrm{U}(1)_Y$ forms a Lie superalgebra.[4] Thus we can define the higher spin algebra $hs(2,2|4)$ as the following coset:[5]

$$hs(2,2|4) = (\mathcal{G}\setminus\mathrm{U}(1)_Y)/\mathcal{I}\,. \tag{2.13}$$

The elements of $hs(2,2|4)$ are thus equivalence classes $[P]$ of elements in $\mathcal{G}$ defined by

$$[P] = \{Q\in\mathcal{G}\mid P-Q\in\mathcal{I}\}\,. \tag{2.14}$$

The Lie bracket of $[P]$ and $[Q]$ is given by

$$[[P],[Q]] = [[P,Q]_*]\,. \tag{2.15}$$

[4]To see this, we need to show that the generator Y never arises on the right hand side of any two commutators in $\mathcal{G}$. Since Y is an $\mathrm{SU}(2,2)\times\mathrm{SU}(4)$ singlet it can only arise on the right hand side of graded commutators $[P_1,\bar{P}_2]$, where P_1 is an arbitrary element of $\mathcal{G}$ and $\bar{P}_2$ its conjugate (P_1 and P_2 have different parameters). There is no loss of generality in replacing P_1 by $F = \bar{y}^{\alpha_1}\cdots\bar{y}^{\alpha_m}y_{\beta_1}\cdots y_{\beta_n}\bar{\theta}^{i_1}\cdots\bar{\theta}^{i_p}\theta_{j_1}\cdots\theta_{j_q}$ (including traces). Computing the part of $[F,F^\dagger]$ which is quadratic in oscillators, one finds that the contributions to K and X is given by an overall sign and combinatorial factor times $(m-n)K+(q-p)X = 2(m-n)Z$, where we have used the τ_η-condition which sets $m-n = q-p$. This shows that $\mathcal{G}\setminus\mathrm{U}(1)_Y$ is a Lie superalgebra on which $\mathrm{U}(1)_Y$ acts as an outer automorphism.

[5]Modding out the ideal generated by $Z-\frac{1}{2}N$, where N is an integer, yields a higher spin extensions of $\mathrm{SU}(2,2|4)$ with central charge $\frac{1}{2}N$ [10].

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In order to exhibit the $\mathrm{SU}(4) \times \mathrm{U}(1)_Y$ content of the algebra we define

$$[K, P] = \Delta P\,, \qquad [X, P] = YP\,, \tag{2.16}$$

where Δ and Y, which is the $\mathrm{U}(1)_Y$ charge, are integers given by

$$\Delta = n_{\bar{y}} - n_y\,, \qquad Y = n_{\bar{\theta}} - n_\theta\,, \tag{2.17}$$

with $n_{\bar{y}} = m$, $n_y = n$, $n_{\bar{\theta}} = p$ and $n_\theta = q$ defined by the oscillator expansion given in (2.5). The condition (2.3) implies that the total $\mathrm{U}(1)_Z$ charge vanishes:

$$2Z = \Delta + Y = 0\,. \tag{2.18}$$

The $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ spins of a generator are given by $(j_L, j_R) = (\frac{1}{2}n_{\bar{y}}, \frac{1}{2}n_y)$. The total spin s of the gauge field associated with the generator in (2.5) is given by $s = 1 + \frac{m+n}{2}$. From τ_η-condition given in (3) it follows that the $\mathrm{SU}(4) \times \mathrm{U}(1)_Y$ content of the higher spin algebra is given by:

$$\begin{array}{l} m+n=0 \\ s=1 \end{array} : \begin{cases} (0,0)X^3 & Y=0 \\ (1,1)(1,X^2) & Y=0 \end{cases}$$

$$\begin{array}{l} m+n=1 \\ s=\dfrac{3}{2} \end{array} : \begin{cases} (1,0)(1,X^2) & Y=1 \\ (2,1)X & Y=1 \end{cases}$$

$$\begin{array}{l} m+n=2 \\ s=2 \end{array} : \begin{cases} (0,0)(1,X^2,X^4) & Y=0 \\ (1,1)X & Y=0 \\ (2,2) & Y=0 \\ (2,0)X & Y=2 \\ (3,1) & Y=2 \end{cases}$$

$$\begin{array}{l} m+n=3,7,11,\ldots \\ s=\dfrac{5}{2},\dfrac{9}{2},\dfrac{13}{2},\ldots \end{array} : \begin{cases} (1,0)(X,X^3) & Y=1 \\ (2,1) & Y=1 \\ (3,0) & Y=3 \end{cases}$$

$$\begin{array}{l} m+n=4,8,12,\ldots \\ s=3,5,7,\ldots \end{array} : \begin{cases} (0,0)(X,X^3) & Y=0 \\ (1,1)(1,X^2) & Y=0 \\ (2,0)(1,X^2) & Y=2 \end{cases}$$

$$\begin{array}{l} m+n=5,9,11\ldots \\ s=\dfrac{7}{2},\dfrac{11}{2},\dfrac{15}{2},\ldots \end{array} : \begin{cases} (1,0)(1,X^2) & Y=1 \\ (2,1)X & Y=1 \\ (3,0)X & Y=3 \end{cases}$$

$\ell \backslash s$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	15	4	1									
1	$16'$	24	36	24	$16'$	4	1					
2			1	4	$16'$	24	36	24	$16'$	4	1	
3							1	4	$16'$	24	36	$\cdots$
4											1	$\cdots$
$\vdots$												

Table 4: The $hs(2,2|4)$ generators with $Y = 0, \pm 1$. The entries are $\mathrm{SU}(4) \times \mathrm{U}(1)_Y$ representations as follows: $16' = 15_0 + 1_0$, $24 = 20_1 + 4_1$, $36 = 20'_0 + 15_0 + 1_0$. These generators are associated with the physical gauge fields all of which have their associated Weyl tensors. The spin s defined by $s = 1 + \frac{1}{2}(n_y + n_{\bar{y}})$ is that of the gauge field associated with the generator. The level ℓ is defined by $\ell = \frac{1}{4}(n_y + n_{\bar{y}} + n_\theta + n_{\bar{\theta}} - 2)$.

$\ell \backslash s$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
1	16	4	6							
2			6	4	$16+1$	4	6			
3							6	4	$16+1$	$\cdots$
$\vdots$										

Table 5: The $hs(2,2|4)$ generators with $Y = \pm 2, \pm 3, \pm 4$. The entries are $\mathrm{SU}(4) \times \mathrm{U}(1)_Y$ representations as follows: $16 = 10_2 + 6_2$, 6_2, 4_3 and 1_4. These generators are associated with gauge fields dual to generalized anti-symmetric tensor fields contained in the scalar master field Φ; see table 6 for $s \geq 1$. Further notation is defined in table 4.

$$\begin{array}{l} m+n = 6, 10, 14, \ldots \\ s = 4, 6, 8, \ldots \end{array} : \begin{cases} (0,0)(1, X^2, X^4) & Y = 0 \\ (1,1)X & Y = 0 \\ (2,2) & Y = 0 \\ (4,0) & Y = 4 \end{cases} \tag{2.19}$$

and hermitian conjugates. Here (p,q) denotes the traceless product of p $\bar{\theta}$'s and q θ's. In deriving this result we have used

$$\left(\bar{\theta}^{i_1} \cdots \bar{\theta}^{i_p} \theta_{i_1} \cdots \theta_{i_q} - \text{trace}\right) X^r = 0 \quad \text{for} \quad p + q + r > 4\,. \tag{2.20}$$

The content of (2.19) is summarized in tables 4 and 5.

3. The spectrum of massless fields

We seek an appropriate massless representation $\mathcal{S}$ of $hs(2,2|4)$ which will be the spectrum of physical fields in a field theoretical realization of $hs(2,2|4)$ in five dimensions. To begin with we observe that each superdoubleton is a representation of

$\mathcal{G}$. Since all massless representations of the $N = 8$ AdS_5 superalgebra consist of the irreps arising in the tensor products of two superdoubletons, we shall assume that all massless irreps of $\mathcal{G}$ also arise in this way. Since we furthermore have defined the higher spin algebra $hs(2,2|4)$ by modding out the central charge Z, the spectrum $\mathcal{S}$ must consist of tensor products between two conjugate spin j superdoubletons with central charges $\frac{1}{2}j$ and $-\frac{1}{2}j$, respectively. We next recall the arguments given in [3, 4, 1], which suggest that the $hs(2,2|4)$ theory arises in the near horizon region of weakly coupled, coinciding D3-branes in the large N limit (i.e. in the limit of tensionless type IIB string theory). If we assume that the higher spin symmetry is realized as a global conformal symmetry on the doubletons in the boundary as well, then it is natural to choose the spectrum $\mathcal{S}$ to be the symmetric tensor product of two Yang-Mills superdoubletons (since these are the only doubletons with $Z = 0$), i.e.

$$\mathcal{S} = (\mathcal{D} \otimes \mathcal{D})_{\mathrm{S}}\,, \tag{3.1}$$

where $\mathcal{D}$ denotes the Yang-Mills superdoubleton:

$$\begin{aligned} \mathcal{D} = [D(0,0;1) \otimes 6]_0 \oplus \left[D\left(\frac{1}{2},0;\frac{3}{2}\right) \otimes 4\right]_{-1} \oplus \left[D\left(0,\frac{1}{2};\frac{3}{2}\right) \otimes \bar{4}\right]_1 \oplus \\ \oplus [D(1,0;2) \otimes 1]_{-2} \oplus [D(0,1;2) \otimes 1]_2\,. \end{aligned} \tag{3.2}$$

The spectrum $\mathcal{S}$ is listed in table 3. The anti-symmetric tensor product gives rise to descendants from the boundary CFT point of view. In fact, the spectrum of the five-dimensional higher spin gauge theory based on $hs(2,2|4)$ is in one-to-one correspondence with the algebra of bilinear supercurrents of the (free) $d = 4$, $N = 4$ Yang-Mills theory (with global $\mathrm{SU}(N)$ symmetry). In this case the right hand side of (3.1) is understood to contain a trace over the adjoint $\mathrm{SU}(N)$ index.

4. Gauging $hs(2,2|4)$

In order to realize $hs(2,2|4)$ as a local symmetry in a field theory with spectrum $\mathcal{S}$ we need to address the issues of auxiliary gauge fields as well as the incorporation of the physical spin $s < 1$ fields and generalized higher spin anti-symmetric tensor fields (which describe the $|Y| > 1$ sector of the spectrum exhibited in table 5).

Gauging of $hs(2,2|4)$ introduces both dynamic gauge fields and auxiliary gauge fields. The structure of the set of gauge fields and curvature constraints, that give rise to one massless spin s degree of freedom on-shell, are known at the linearized level (in an expansion around AdS_5 spacetime). These were first given using tensors and tensor-spinors [14, 15]. In [1] this structure was converted into the multi-spinor basis in the case of $\Delta = 0$ (i.e. bosonic gauge fields). This has an obvious generalization to $|\Delta| = 1$ (i.e. fermionic gauge fields), which we shall give below. The gauge fields corresponding to the generators listed in table 4 (having $|Y| = |\Delta| \leq 1$) therefore give rise to physical spin s degrees of freedom.

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$\ell \backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	42	48	$\underline{6}$											
1	1	8	$\underline{6}$	$\underline{4}$	$16+\underline{1}$	4	6							
2							6	4	$16+1$	4	6			
3											6	4	$16+1$	$\cdots$
$\vdots$														

Table 6: The physical matter fields contained in the master scalar field. The entries are the following $\mathrm{SU}(4) \times \mathrm{U}(1)_Y$ representations for $s < 1$: $42 = 20'_0 + 10_2 + \bar{10}_{-2} + 1_4 + \bar{1}_{-4}$, $48 = 20_1 + \bar{20}_{-1} + 4_3 + \bar{4}_{-3}$, $8 = 4_1 + \bar{4}_{-1}$ and 1_0; for $s \geq 1$: 6_2, 4_3, $16 = 10_2 + 6_2$ and 1_4. The spin $s \geq 1$ sector is realized in the field theory in terms of generalizations of the anti-symmetric two-form potential. These fields obey self-duality in $D = 5$ and have dual one-form gauge fields corresponding to the generators given in table 5, with the exception of the underlined representations, which have no one-form duals. Here the form degree refers to the number of curved indices as opposed to the tangential multi-spinor indices arising from the $(y, \bar{y})$-expansion.

The generators listed in table 5 (having $|\Delta| = |Y| > 1$) do not give rise to the canonical set of spin s physical and auxiliary gauge fields. As we shall see these gauge fields instead have the interpretation of higher spin duals of a certain tower of higher spin generalizations of the anti-symmetric two-form potential that arises in the supergravity multiplet. Thus, comparing the spectrum $\mathcal{S}$ given in table 3 with the physical states arising from the algebra, as listed in table 4, we find that the spectrum has a 'matter content' (the spin $s < 1$ sector and the above mentioned two-form potentials), given by the states listed in table 6, which cannot be realized using the gauge field.

In order to accommodate the spectrum $\mathcal{S}$ in a field theoretical construction in five dimensions we introduce a master gauge field $[A]$ in the adjoint representation of $hs(2,2|4)$, where $A = dx^\mu A_\mu$ is a $\mathcal{G}$-valued one-form, and a master scalar field Φ in a quasi-adjoint representation $\mathcal{R}$ of $hs(2,2|4)$ defined as follows ($|\eta| = 1$):

$$\tau_\eta(A) = -A\,, \qquad (A)^\dagger = -A\,, \tag{4.1}$$

$$\tau_\eta(\Phi) = \pi_\eta(\Phi)\,, \quad (\Phi)^\dagger = \pi_{-i}(\Phi)\,, \tag{4.2}$$

$$Z \star \Phi = \Phi \star \pi_{-i}(Z) = 0\,, \tag{4.3}$$

where the anti-involution τ_η is defined in (2.1) and π_η is the involution ($|\eta| = 1$)

$$\pi_\eta(F(y_\alpha, \bar{y}^{\dot\alpha}, \theta_i, \bar{\theta}^i)) = F(-i\bar{\eta}\bar{y}_{\dot\alpha}, i\eta y^\alpha, i\eta\theta_i, -i\bar{\eta}\bar{\theta}^i)\,, \tag{4.4}$$

$$\pi_\eta(F \star G) = \pi_\eta(F) \star \pi_\eta(G)\,. \tag{4.5}$$

The $\mathcal{G}$ gauge transformations are given by:

$$\delta_\epsilon A = d\epsilon + [A, \epsilon]_*\,, \tag{4.6}$$

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$$\delta_\epsilon \Phi = \Phi \star \pi_{-i}(\epsilon) - \epsilon \star \Phi\,, \tag{4.7}$$

where ϵ is a $\mathcal{G}$ valued local parameter. The following curvature and covariant derivative obey (4.1) and (4.2)

$$F_A = dA + A \wedge \star A\,, \tag{4.8}$$

$$D_A \Phi = d\Phi - \Phi \star \pi_{-i}(A) + A \star \Phi\,, \tag{4.9}$$

and transform in a $\mathcal{G}$-covariant way as

$$\delta_\epsilon F_A = [F_A, \epsilon]_\star\,, \tag{4.10}$$

$$\delta_\epsilon D_A \Phi = D_A \Phi \star \pi_{-i}(\epsilon) - \epsilon \star D_A \Phi\,. \tag{4.11}$$

The π_{-i}-maps are inserted in the definitions of $\delta_\epsilon \Phi$ and $D_A \Phi$ to ensure that they obey the τ-covariance and reality conditions given in (4.2). To show this we first note that an element $P \in \mathcal{G}$ obeys

$$\pi_\eta^{-1}(P) = \pi_{-i}(P)\,, \qquad \tau_\eta(\pi_{-i}(P)) = -\pi_\eta(P)\,, \tag{4.12}$$

which can be checked explicitly. From this it follows that

$$\begin{aligned}\tau_\eta\left[\Phi \star \pi_{-i}(P) - P \star \Phi\right] &= \tau_\eta(\pi_{-i}(P)) \star \tau_\eta(\Phi) - \tau_\eta(\Phi) \star \tau_\eta(P) \\ &= -\pi_\eta(P) \star \pi_\eta(\Phi) + \pi_\eta(\Phi) \star P = \pi_\eta\left[\Phi \star \pi_{-i}(P) - P \star \Phi\right],\end{aligned}$$

hence the τ-covariance of $\delta_\epsilon \Phi$ and $D_A \Phi$. To check the reality condition, we first note that $\pi_{-i}^2 = 1$, which implies that $(\pi_{-i}(F))^\dagger = \pi_{-i}(F^\dagger)$ for any function F of the oscillators. It then follows that

$$\begin{aligned}\left[\Phi \star \pi_{-i}(P) - P \star \Phi\right]^\dagger &= (\pi_{-i}(P))^\dagger \star \Phi^\dagger - \Phi^\dagger \star P^\dagger \\ &= -\pi_{-i}(P) \star \pi_{-i}(\Phi) + \pi_{-i}(\Phi) \star P = \pi_{-i}\left[\Phi \star \pi_{-i}(P) - P \star \Phi\right],\end{aligned}$$

hence the reality conditions of $\delta_\epsilon \Phi$ and $D_A \Phi$. The $hs(2,2|4)$ gauge field, curvature, covariant derivative and gauge parameter are defined by

$$[A]\,, \qquad F_{[A]} = [F_A]\,, \qquad D_{[A]}\Phi = D_A \Phi\,, \qquad [\epsilon]\,, \tag{4.13}$$

where we use the notation defined in (2.14). The $hs(2,2|4)$ gauge transformations are

$$\delta_{[\epsilon]}[A] = [\delta_\epsilon A] \qquad \delta_{[\epsilon]} F_{[A]} = [\delta_\epsilon F_A]\,, \qquad \delta_{[\epsilon]}\Phi = \delta_\epsilon \Phi\,. \tag{4.14}$$

The condition (4.3) implies that $D_A \Phi$ and $\delta_\epsilon \Phi$ are independent of the choice of $\mathcal{G}$-valued representatives A and ϵ. The curvature $F_{[A]}$ and the gauge transformation

$\delta_{[\epsilon]}[A]$ are computed by first evaluating the ordinary $\star$ product between the representatives and then expanding the result with respect to the particular ordering of oscillators defined by (2.8) and finally discarding any terms in $\mathcal{I}$. In case one would have to perform several repeated multiplications of objects in $hs(2,2|4)$ the last step may of course be carried out at the end, as the operation of modding out Z commutes with taking the $\star$ product.

As found in the previous section, the gauge field $[A]$ can be represented by $A \in \mathcal{G}^{(0)}$, which has an expansion in terms of component fields $A_\mu{}_{\alpha_1\ldots\alpha_m}^{\beta_1\ldots\beta_n}{}_{i_1\ldots i_p}^{j_1\ldots j_q}$, with tangent indices corresponding to the generators given in (2.5) and (2.6). The SU(4) content is listed explicitly in (2.19); see also tables 4 and 5.

5. The field content of Φ

The field content of the master field Φ is determined by the τ_η and reality conditions in (4.2) and the Z-condition in (4.3). To solve these conditions it is convenient to use the notation:

$$\Phi^{(r;t)}(m,n;p,q) = \frac{1}{m!n!p!q!}\Phi^{(r;t)}{}_{\alpha_1\ldots\alpha_m}^{\beta_1\ldots\beta_n}{}_{i_1\ldots i_p}^{j_1\ldots j_q}\bar{y}^{\alpha_1}\cdots\bar{y}^{\alpha_m}y_{\beta_1}\cdots y_{\beta_n}\bar{\theta}^{i_1}\cdots\bar{\theta}^{i_p}\theta_{j_1}\cdots\theta_{j_q}\,, \tag{5.1}$$

where the superscript r denotes the number of anti-symmetric pairs of spinor indices and t ($0 \le t \le r$) the number of these that are traced. The spin of a component field of Φ is defined by $s = \frac{m+n}{2} - t$. The relation between the labels s and r and the SO(5) highest weights (m_1, m_2) ($m_1 \ge m_2 \ge 0$) of the irreducible component $T^{(m_1,m_2)}$ contained in $\Phi^{(r;t)}$ are given in (B.6). Below we shall most frequently use the following components:

$$\begin{aligned}
\Phi^{(0;0)}_{\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} &= T^{(\frac{m+n}{2},\frac{m+n}{2})}_{\alpha_1\ldots\alpha_m\beta_1\ldots\beta_n}\,,\\
\Phi^{(1;0)}_{\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} &= T^{(\frac{m+n}{2},\frac{m+n-2}{2})}_{\alpha_1\ldots\alpha_m\beta_1\ldots\beta_n}\,,\\
\Phi^{(1;1)}_{\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} &= C_{\alpha_1\beta_1}T^{(\frac{m+n-2}{2},\frac{m+n-2}{2})}_{\alpha_2\ldots\alpha_m\beta_1\ldots\beta_n}\,,\\
\Phi^{(2;0)}_{\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} &= T^{(\frac{m+n}{2},\frac{m+n-4}{2})}_{\alpha_1\ldots\alpha_m\beta_1\ldots\beta_n}\,,\\
\Phi^{(2;1)}_{\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} &= C_{\alpha_1\beta_1}T^{(\frac{m+n-2}{2},\frac{m+n-4}{2})}_{\alpha_2\ldots\alpha_m\beta_1\ldots\beta_n}\,,\\
\Phi^{(2;2)}_{\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} &= C_{\alpha_1\beta_1}C_{\alpha_2\beta_2}T^{(\frac{m+n-4}{2},\frac{m+n-4}{2})}_{\alpha_3\ldots\alpha_m\beta_1\ldots\beta_n}\,,
\end{aligned}$$

where the separate symmetrizations in α and β-indices on the right hand sides have been suppressed. The τ_η-condition implies

$$m+n+p+q = \begin{cases} 0 \bmod 2 & \text{for } m \neq n \\ 2r \bmod 4 & \text{for } m = n \end{cases}. \tag{5.2}$$

From this it follows that if $\Phi^{(0;0)}(m,n;p,q)$ is allowed, then $\Phi^{(r;t)}(m+r,n+r;p,q)$n with $0\le t\le r$ are also allowed. It is therefore sufficient to analyze the SU(4) content for $r=t=0$. Using (2.20), we find that for $m=n$ the possible SU(4) contents are ($s=\frac{m+n}{2}$):

$$\begin{array}{l} m=n=0,2,\ldots \\ s=0,2,\ldots \end{array} : \begin{cases} (0,0)(1,X^2,X^4) \\ (2,2) \\ (1,1)X \\ (3,1) \\ (2,0)X \\ (4,0) \end{cases}$$
$$\begin{array}{l} m=n=1,3,\ldots \\ s=1,3,\ldots \end{array} : \begin{cases} (0,0)(X,X^3) \\ (1,1)(1,X^2) \\ (2,0)(1,X^2) \end{cases} \tag{5.3}$$

and hermitian conjugates, where (p,q) denotes the traceless product of p number of $\bar\theta$'s and q number of θ's. For $m\neq n$ we find

$$\begin{array}{l} m+n=1,3,\ldots \\ s=\dfrac{1}{2},\dfrac{3}{2},\ldots \end{array} : \begin{cases} (1,0)(1,X,X^2,X^3) \\ (2,1)(1,X) \\ (3,0)(1,X) \end{cases}$$
$$\begin{array}{l} m+n=2,4,\ldots \\ s=1,2,\ldots \end{array} : \begin{cases} (0,0)(1,X,X^2,X^3,X^4) \\ (1,1)(1,X,X^2) \\ (2,0)(1,X,X^2) \\ (4,0) \\ (3,1) \\ (2,2) \end{cases} \tag{5.4}$$

and hermitian conjugates. The reality condition in (4.2) determines $\Phi(m,n;p,q)$ in terms of $\Phi(n,m;p,q)$.

We next examine the Z-condition (4.3). Adding and subtracting the two equations, we obtain

$$\{X,\Phi(m,n;p,q)\}_\star+\Delta\Phi(m,n;p,q)=0\,, \tag{5.5}$$
$$\{K,\Phi(m,n;p,q)\}_\star+Y\Phi(m,n;p,q)=0\,, \tag{5.6}$$

where $\Delta=m-n$ and $Y=p-q$, which defines the $\mathrm{U}(1)_Y$ charge.[6] The anti-commutators in (5.5) and (5.6) are evaluated using ($t=0,1,\ldots$; $u=0,\ldots,4$)

$$K\star\big(K^tX^uT(m,n;p,q)\big)=\left(K^{t+1}+\frac{1}{2}\Delta K^t-\frac{1}{4}t(t+3+m+n)K^{t-1}\right)\times$$

[6]The $\mathrm{U}(1)_Y$-charge of Φ is given by $Y\Phi:=X\star\Phi-\Phi\star\pi_{-i}(X)=[X,\Phi]_\star$.

$$
\begin{aligned}
&\times X^u T(m,n;p,q)\,,\\
\big(K^t X^u T(m,n;p,q)\big)\star K &= \left(K^{t+1}-\frac{1}{2}\Delta K^t-\frac{1}{4}t(t+3+m+n)K^{t-1}\right)\times\\
&\times X^u T(m,n;p,q)\,,\\
X\star\big(K^t X^u T(m,n;p,q)\big) &= \left(X^{u+1}+\frac{1}{2}YX^u-\frac{1}{4}u(u-5+p+q)X^{u-1}\right)\times\\
&\times K^t T(m,n;p,q)\,,\\
\big(K^t X^u T(m,n;p,q)\big)\star X &= \left(X^{u+1}-\frac{1}{2}YX^u-\frac{1}{4}u(u-5+p+q)X^{u-1}\right)\times\\
&\times K^t T(m,n;p,q)\,,
\end{aligned}
$$

where $T(m,n;p,q)$ is assumed to be traceless both in SU(2,2) and SU(4) indices. Eq. (5.5) gives a characteristic equation for Δ and furthermore eliminates some of the SU(4) representations listed in (5.3) and (5.4). We find ($Y=p-q$ for $(p,q)X^r$ elements; $\Delta=m-n$):

$$
\begin{array}{l}
m=n=0,2,\ldots\\
s=0,2,\ldots
\end{array}
:\left\{
\begin{array}{l}
(0,0)\left(1-\frac{2}{3}X^2+\frac{2}{3}X^4\right)\\
(2,2)\\
(3,1)\\
(4,0)
\end{array}\right.
$$

$$
\begin{array}{l}
m=n=1,3,\ldots\\
s=1,3,\ldots
\end{array}
:\left\{
\begin{array}{l}
(1,1)\left(1-\frac{1}{2}X^2\right)\\
(2,0)\left(1-\frac{1}{2}X^2\right)
\end{array}\right.
$$

$$
\begin{array}{l}
m+n=1,3,\ldots\\
s=\frac{1}{2},\frac{3}{2},\ldots
\end{array}
:\left\{
\begin{array}{ll}
(1,0)\left(\begin{array}{c}1-\frac{2\Delta}{3}X+\frac{1}{3}(\Delta^2-3)X^2\\ -\frac{2}{3\Delta}(\Delta^2-3)X^3\end{array}\right) & \Delta=\pm1,\pm3\\
(2,1)(1-2\Delta X) & \Delta=\pm1\\
(3,0)(1-2\Delta X) & \Delta=\pm1
\end{array}\right.
$$

$$
\begin{array}{l}
m+n=2,4,\ldots\\
s=1,2,\ldots
\end{array}
:\left\{
\begin{array}{ll}
(0,0)\left(\begin{array}{c}1-\frac{\Delta}{2}X+\frac{1}{6}(\Delta^2-4)X^2\\ -\frac{\Delta}{18}(\Delta^2-10)\left(1-\frac{2}{\Delta}X\right)X^3\end{array}\right) & \Delta=\pm2,\pm4\\
(1,1)(1-\Delta X+2X^2) & \Delta=\pm2\\
(2,0)(1-\Delta X+2X^2) & \Delta=\pm2
\end{array}\right.
\tag{5.7}
$$

and hermitian conjugates (which have opposite values of Y). The condition (5.6) relates components of Φ that have different powers of K. Denote such structures by

$\Phi_t K^t = \Phi^{(r+t;t)}(m,n;p,q)$ and expand

$$\Phi(K) = \sum_{t=0}^{\infty} \Phi_t K^t \,. \tag{5.8}$$

From (5.6) it follows that

$$2\Phi_{t-1} + Y\Phi_t - \frac{t+1}{2}(t+4+m+n)\Phi_{t+1} = 0 \,. \tag{5.9}$$

This equation determines Φ_t uniquely in terms of the leading coefficient Φ_0:

$$\Phi(K) = f(m+n, Y; K)\Phi_0 \,, \tag{5.10}$$

where the function $f(m+n, Y; z)$ is an analytic function. In the analysis of the linearized field equations one only needs to expand up to first order in K, except in the scalar sector where also the second order is needed. The first order coefficients are given by

$$\Phi^{(r+1;1)}_{\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n}(\theta,\bar\theta) = Y\frac{mn}{m+n+2} C_{\alpha_1\beta_1}\Phi^{(r;0)}_{\alpha_2\ldots\alpha_m,\beta_2\ldots\beta_n}(\theta,\bar\theta) \,, \tag{5.11}$$

where separate symmetrization of the α and β indices is understood. In the case of scalars the K-expansion reads:

$$\phi(K) = \left(1 + \frac{Y}{2}K + \left(\frac{2}{5} + \frac{Y^2}{10}\right) K^2 + \cdots\right)\phi_0 \,. \tag{5.12}$$

In summary, the field content of Φ is arranged into K and X expansions starting with $SU(2,2)$ traceless multi-spinors $\Phi^{(r;0)}(m,n;p,q)$ whose $SU(4)$ content is given by (5.7). As we shall see in the next section, the expansion of the anti-symmetric traceless $SU(2,2)$ indices yields trajectories of the schematic form

$$\Phi^{(r;0)}(m,n;p,q) \sim P^{a_1}\cdots P^{a_r}\nabla_{a_1}\cdots\nabla_{a_r}\Phi^{(0;0)}(m-r,n-r;p,q) \,,$$

so that only $\Phi^{(0;0)}(m,n;p,q)$ are independent fields. The sector with $s \geq 1$ and $|Y| \leq 1$ are Weyl tensors while the sector with $s \geq 1$ and $|Y| > 1$ are independent three-form field strengths (carrying internal tangent space indices when $s > 1$) obeying self-duality equations in $D = 5$. The $s < 1$ sector defines physical scalars and fermions. Thus the independent field content of Φ matches that of the spectrum in table 3 (though it remains to verify that the lowest energies satisfy (1.5)). By construction, the components of Φ fall into supermultiplets labeled by the level index of (2.5):

$$\begin{aligned} \Phi &= \sum_{\ell=0,1,2,\ldots} \Phi_{(\ell)} \,, \\ \delta_\epsilon \Phi_{(\ell)} &= \epsilon \star \Phi_{(\ell)} - \Phi_{(\ell)} \star \pi_{-i}(\epsilon) \,, \end{aligned} \tag{5.13}$$

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where $\epsilon = \epsilon^i_\alpha \bar{y}^\alpha \theta_i - h.c$ denotes the supersymmetry parameter. The level index breaks the degeneracy whenever there are several components of Φ carrying the same $\mathrm{SU}(2,2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ representation.

The spin $s \leq 1$ sector is embedded in Φ as follows:

$$\begin{aligned}
\Phi = {} & \phi(\theta,\bar{\theta};K) + y^\alpha \lambda_\alpha(\theta,\bar{\theta};K) + \bar{y}^\alpha \tilde{\lambda}_\alpha(\theta,\bar{\theta};K) + \\
& + \frac{1}{2} y^\alpha y^\beta \phi_{\alpha\beta}(\theta,\bar{\theta};K) + \frac{1}{2} \bar{y}^\alpha \bar{y}^\beta \tilde{\phi}_{\alpha\beta}(\theta,\bar{\theta};K) + \\
& + y^{(\alpha} \bar{y}^{\beta)} G_{\alpha\beta}(\theta,\bar{\theta};K) + i\bar{y}\Gamma^a y \phi_a(\theta,\bar{\theta};K) + \cdots ,
\end{aligned} \tag{5.14}$$

where

$$\begin{aligned}
\phi(\theta,\bar{\theta};K) = {} & \frac{1}{4}\bar{\theta}^i\bar{\theta}^j\theta_k\theta_l\phi^{kl}_{ij}(K) + \frac{1}{6}\bar{\theta}^i\bar{\theta}^j\bar{\theta}^k\theta_l\phi^l_{ijk}(K) + \frac{1}{24}\bar{\theta}^i\bar{\theta}^j\bar{\theta}^k\bar{\theta}^l\phi_{ijkl}(K) + \\
& + \left(1 - \frac{2}{3}X^2 + \frac{2}{3}X^4\right)\phi(K) + \text{conj.} , \\
\lambda_\alpha(\theta,\bar{\theta};K) = {} & (1+2X)\left[\frac{1}{2}\bar{\theta}^i\bar{\theta}^j\theta_k\lambda_{\alpha}{}^k_{ij}(K) + \frac{1}{6}\bar{\theta}^i\bar{\theta}^j\bar{\theta}^k\lambda_{\alpha ijk}(K)\right] + \\
& + \left(1 + \frac{2}{3}X - \frac{2}{3}X^2 - \frac{4}{3}X^3\right)\bar{\theta}^i\lambda_{\alpha\, i}(K) + \text{conj.} , \\
\phi_{\alpha\beta}(\theta,\bar{\theta};K) = {} & \frac{1}{2}(\Gamma^{ab})_{\alpha\beta}\Bigg[(1+2X+2X^2)\bar{\theta}^i\theta_j F_{ab}{}^j_i(K) + \\
& \qquad + \left(1 + 2X - \frac{2}{3}X^3 - \frac{2}{3}X^4\right)F_{ab}(K)\Bigg] + \\
& + \frac{1}{12}(\Gamma^{abc})_{\alpha\beta}(1+2X+2X^2)\bar{\theta}^i\bar{\theta}^j H_{abc\, ij}(K) + \text{conj.} , \\
G_{\alpha\beta}(\theta,\bar{\theta};K) = {} & \left(1 - \frac{1}{2}X^2\right) \times \\
& \times \left[\frac{1}{2}(\Gamma^{ab})_{\alpha\beta}\bar{\theta}^i\theta_j F'_{ab}{}^j_i(K) + \frac{1}{12}(\Gamma^{abc})_{\alpha\beta}\bar{\theta}^i\bar{\theta}^j H'_{abc\, ij}(K)\right] + \text{conj.} .
\end{aligned} \tag{5.15}$$

Here all SU(4) representations are irreducible and the K-dependence, which is of the form (5.10), is determined by (5.9). The conjugates are determined by the reality conditions:

$$\begin{aligned}
(\phi(\theta,\bar{\theta}))^\dagger &= \phi(\theta,\bar{\theta}) , & (\lambda_\alpha(\theta,\bar{\theta}))^\dagger &= -\bar{\lambda}_\alpha(\theta,\bar{\theta}) , \\
(\phi_{\alpha\beta}(\theta,\bar{\theta}))^\dagger &= \bar{\phi}_{\alpha\beta}(\theta,\bar{\theta})^\dagger , & (G_{\alpha\beta}(\theta,\bar{\theta}))^\dagger &= \bar{G}_{\alpha\beta}(\theta,\bar{\theta}) ,
\end{aligned} \tag{5.16}$$

where $\bar{\lambda}_\alpha$, $\bar{\phi}_{\alpha\beta}$ and $\bar{G}_{\alpha\beta}$ are defined by (2.10). The τ_η-condition implies $\tilde{\lambda}_\alpha(\theta,\bar{\theta}) = i\lambda_\alpha(i\theta, i\bar{\theta})$ and $\tilde{\phi}_{\alpha\beta}(\theta,\bar{\theta}) = -\phi_{\alpha\beta}(i\theta, i\bar{\theta})$. The SU(4) content and reality condition of $\phi_a(\theta,\bar{\theta})$ is the same as that of $\phi(\theta,\bar{\theta})$. The linearized field equation will set $\phi_a(\theta,\bar{\theta}) \sim \partial_a\phi(\theta,\bar{\theta})$ and equate $F_{ab}{}^j_i$, $F'_{ab}{}^j_i$ and F_{ab} with the curvatures of the corresponding spin

1 gauge fields, while $H_{abc\,ij}$ and $H'_{abc\,ij}$ will be field strengths of physical two-form potentials $B_{\mu\nu\,ij}$ and $B'_{\mu\nu\,ij}$. The level $\ell = 1$ fields are ϕ, $\lambda_{\alpha i}$, F_{ab}, linear combinations of $F_{ab}{}_i^j$ and $F'_{ab}{}_i^j$, and linear combinations of $H_{abc\,ij}$ and $H'_{abc\,ij}$, determined by (5.13).

6. The linearized constraints and their integrability

We assume that the higher spin gauge theory can be expanded around the AdS-vacuum described by $\Phi = 0$ and $A = \Omega$. Building upon the bosonic results in [1], we propose the following linearized constraints:

$$
\begin{aligned}
F_{(\ell)\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n}{}_{i_1\ldots i_p}^{j_1\ldots j_q} &= e^a \wedge e^b (\Gamma_{ab})^{\gamma\delta} \Phi^{(0;0)}_{(\ell)\gamma\alpha_1\ldots\alpha_m,\delta\beta_1\ldots\beta_n}{}_{i_1\ldots i_p}^{j_1\ldots j_q}\,|_{\mathcal{G}^{(0)}\setminus\,\mathrm{U}(1)_Y}\,, && (6.1)\\
d\Phi + \Omega \star \Phi - \Phi \star \pi_{-i}(\Omega) &= 0\,, && (6.2)
\end{aligned}
$$

where $\Phi^{(0;0)}$ is the fully symmetric part of Φ, as defined in (5.1), and F is the AdS_5 covariant linearized field strength of the $\mathcal{G}^{(0)}$-valued gauge field A:

$$
\begin{aligned}
F &= dA + \Omega \star A + A \star \Omega\,,\\
F_{\mu\nu\,\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} &= 2\nabla_{[\mu} A_{\nu]\,\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} + \\
&\quad + m(\Gamma_{[\mu})_{\alpha_1}{}^{\gamma} A_{\nu]\,\gamma\alpha_2\ldots\alpha_m,\beta_1\ldots\beta_n} - n(\Gamma_{[\mu})_{\beta_1}{}^{\gamma} A_{\nu]\,\alpha_1\ldots\alpha_m,\gamma\beta_2\ldots\beta_n}\,. \qquad (6.3)
\end{aligned}
$$

Here ∇_μ is the background Lorentz covariant derivative, SU(4) indices are suppressed and separate symmetrizations in α and β indices are understood. In (6.1) the subscript $\ell = 0, 1, 2, \ldots$ refers to the supermultiplet level index defined in (2.5) and (5.13), and the SU(4) indices are assumed to be irreducible (which means that p and q do not in general obey the condition in (2.5)). Below, in verifying integrability and finding the linearized field equations, we can suppress both SU(4) and level indices, due to the linear nature of all equations. For $m = n$ and suitable restrictions on the SU(4) content (see [1]), the equations (6.1) and (6.2) are precisely the bosonic equations proposed in [1].

The right hand side of (6.1) is a projection of $\Phi^{(0;0)}$, both in its $(y, \bar{y})$ and $(\theta, \bar{\theta})$-expansion. The components projected out are the $s < 1$ sector and the underlined SU(4) irreps shown in table 6. While this, as well as the matching of K and X-dependence, is done here by hand, we expect that these operations arise naturally in the full interacting theory, as is the case in four dimensions, such that (6.1) can be written as a master constraint $F = \mathcal{V}(\Phi)$, where $\mathcal{V}$ is a map involving the background fünfbein and the generator X. The linearized constraint (6.2), on the other hand, is already in the desired form.

Before we analyze the consequences of (6.1) and (6.2) for the linearized field equations, we first wish to establish their integrability. Using $d\Omega + \Omega \star \Omega = 0$, it immediately follows that (6.2) is integrable. The integrability of (6.1) requires the right hand side to obey the Bianchi identity[7]

$$dF + \Omega \star F - F \star \Omega = 0\,, \tag{6.4}$$

$$\nabla_{[\mu} F_{\nu\rho]\,\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} + \frac{m}{2}(\Gamma_{[\mu})_{\alpha_1}{}^{\gamma} F_{\nu\rho]\,\gamma\alpha_2\ldots\alpha_m,\beta_1\ldots\beta_n} - \frac{n}{2}(\Gamma_{[\mu})_{\beta_1}{}^{\gamma} F_{\nu\rho]\,\alpha_1\ldots\alpha_m,\gamma\beta_2\ldots\beta_n} = 0\,. \tag{6.5}$$

For $m = n$ this was shown in [1]. To show this in general we first write the constraints (6.1) and (6.2) in components as follows:

$$F_{\mu\nu\,\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} = \frac{1}{8}(\Gamma_{\mu\nu})^{\gamma\delta}\Phi^{(0;0)}_{\gamma\alpha_1\ldots\alpha_m,\delta\beta_1\ldots\beta_n}\,, \tag{6.6}$$

$$\nabla_\mu \Phi_{\alpha_1\ldots\alpha_m,\beta_1\ldots\beta_n} = \frac{1}{2}(\Gamma_\mu)^{\gamma\delta}\Phi_{\gamma\alpha_1\ldots\alpha_m,\delta\beta_1\ldots\beta_n} - \frac{mn}{2}(\Gamma_\mu)_{\alpha_1\beta_1}\Phi_{\alpha_2\ldots\alpha_m,\beta_2\ldots\beta_n}\,. \tag{6.7}$$

We next substitute (6.6) into (6.5) and use (6.7). Upon decomposing the free α and β indices into $\mathrm{SU}(2,2)$ irreps, all structures vanish identically except the $(0;0)$ and $(1;0)$ parts, where we use the notation of (5.1). The $(1;0)$ part vanishes due to the following Fierz identity [1]:

$$(\Gamma_{a[\mu})^{\beta\gamma}(\Gamma_{\nu\rho]})^{\delta\epsilon}\Phi^{(0;0)}_{\beta\gamma\delta\epsilon\alpha_1\ldots\alpha_{m+n-2}} = 0\,. \tag{6.8}$$

The $(0;0)$ part of (6.5) takes the form

$$\begin{aligned}&\frac{1}{2}(\Gamma^{\beta\gamma}_{[\mu\nu}(\Gamma_{\rho]})^{\delta\epsilon}\left[\Phi^{(1;0)}_{\beta\gamma\delta\epsilon\alpha_1\ldots\alpha_{m+n}} + \Phi^{(1;1)}_{\beta\gamma\delta\epsilon\alpha_1\ldots\alpha_{m+n}}\right] + \\ &\qquad + \frac{m-n}{2}(\Gamma_{[\mu})_{(\alpha_1}{}^{\beta}(\Gamma_{\nu\rho]})^{\gamma\delta}\Phi^{(0;0)}_{\alpha_2\ldots\alpha_{m+n})\beta\gamma\delta} = 0\,,\end{aligned} \tag{6.9}$$

where $\Phi^{(1;1)}$ is given in terms of $\Phi^{(0;0)}$ by (5.11), with the $\mathrm{U}(1)_Y$ charge $Y = n - m$ (the charge of $\Phi^{(0;0)}$ is the same as that of the field strength, which in turn is given by $Y = -\Delta = n - m$, as follows from (2.18)). The contribution to (6.9) from $\Phi^{(1;0)}$ vanishes by means of the following Fierz identity [1]:

$$(\Gamma^{\beta\gamma}_{[\mu\nu}(\Gamma_{\rho]})^{\delta\epsilon}\Phi^{(1;0)}_{\beta\gamma\delta\epsilon\alpha_1\ldots\alpha_{m+n}} = 0\,. \tag{6.10}$$

Thus, after some algebra,[8] one finds that (6.9) is equivalent to

$$\begin{aligned}Y\Bigg\{\frac{1}{m+n+6}&\left((\Gamma_{\mu\nu\rho]})^{\beta\gamma}\Phi^{(0;0)}_{\beta\gamma\alpha_1\ldots\alpha_{m+n}} + \frac{m+n}{2}(\Gamma_{[\mu})_{(\alpha_1}{}^{\beta}(\Gamma_{\nu\rho]})^{\gamma\delta}\Phi^{(0;0)}_{\alpha_2\ldots\alpha_{m+n})\beta\gamma\delta}\right) - \\ &- \frac{1}{2}(\Gamma_{[\mu})_{(\alpha_1}{}^{\beta}(\Gamma_{\nu\rho]})^{\gamma\delta}\Phi^{(0;0)}_{\alpha_2\ldots\alpha_{m+n})\beta\gamma\delta}\Bigg\} = 0\,,\end{aligned} \tag{6.11}$$

[7] Here and in the remainder of this section the separate symmetrizations in α and β indices is understood.

[8] In order to substitute (5.11) into (6.9) one needs to first split the indices $\alpha_1\ldots\alpha_{m+1}$ and $\beta_1\ldots\beta_{n+1}$ into $\alpha_1\ldots\alpha_m\gamma\epsilon$ and $\alpha_1\ldots\alpha_n\delta\varphi$ and then contract with $(\Gamma_{[\mu})^{\gamma\delta}(\Gamma_{\nu\rho]})^{\epsilon\varphi}$ and finally symmetrize the remaining free indices.

where the first two terms come from $\Phi^{(1;1)}$. This equation simplifies as:[9]

$$Y\left\{(\Gamma_{\mu\nu\rho})^{\beta\gamma}\Phi^{(0;0)}_{\beta\gamma\alpha_1\dots\alpha_{m+n}}-3(\Gamma_{[\mu})_{(\alpha_1}{}^{\beta}(\Gamma_{\nu\rho]})^{\gamma\delta}\Phi^{(0;0)}_{\alpha_2\dots\alpha_{m+n})\beta\gamma\delta}\right\}=0\,. \tag{6.12}$$

This is satisfied due to the following Fierz identity:

$$\left[(\Gamma_{ab})^{\beta\gamma}(\Gamma_{\mu\nu\rho})^{\delta\epsilon}-3(\Gamma_{[\mu\nu})^{\beta\gamma}(\Gamma_{\rho]ab})^{\delta\epsilon}\right]\Phi^{(0;0)}_{\beta\gamma\delta\epsilon\alpha_1\dots\alpha_{m+n-2}}=0\,, \tag{6.13}$$

which follows from the five-dimensional membrane identity:

$$(\Gamma^{\mu\nu})_{(\alpha\beta}(\Gamma_\nu)_{\gamma)\delta}=0\,. \tag{6.14}$$

To apply (6.13) to (6.12), we contract pairs of α-indices by second rank Γ-matrices, and symmetrize all pairs of anti-symmetric vector indices. In the case of odd $m+n$, one in addition has to make use of the Fierz identity

$$(\Gamma^{[\mu})_{[\epsilon}{}^{\beta}(\Gamma^{\nu\rho]})^{\gamma\delta}\Phi^{(0;0)}_{\varphi]\alpha_1\dots\alpha_{m+n-2}\beta\gamma\delta}=0\,,$$

which follows from (6.8).

Thus we have established the integrability of the constraints (6.1) and (6.2) describing the linearization of the $hs(2,2|4)$ gauge theory around AdS_5 spacetime.

7. The linearized field equations

7.1 The master scalar constraint

We begin by analyzing the constraint (6.2) on the master scalar field, since we expect all physical degrees of freedom of the theory to be represented in Φ. The physical spin $s\geq 1$ fields occur in Φ via their field strengths or their derivatives. Interestingly, as we shall see below, a subset of these are three-form field strengths obeying self-duality equations in five dimensions. These are higher spin generalizations of the well-known field equation for the two-form potential in the supergravity multiplet, and yield states in the spectrum with $|Y|>1$ (the remaining spin $s\geq 1$ states, which has $|Y|\leq 1$, occur in Φ via their Weyl tensors).

The master scalar constraint (6.7) implies that $\Phi^{(r;0)}_{\alpha_1\dots\alpha_{2s+2r}}$ $(r\geq 1)$ can be expressed in terms of (up to r) derivatives of $\Phi^{(0;0)}_{\alpha_1\dots\alpha_{2s}}$:

$$(\Gamma_{a_1})^{\alpha_1\alpha_2}\cdots(\Gamma_{a_r})^{\alpha_{2r-1}\alpha_{2r}}\Phi^{(r;0)}_{\alpha_1\dots\alpha_{2r}\beta_1\dots\beta_{2s}}=2^r\nabla_{(a_1}\cdots\nabla_{a_r)}\Phi^{(0;0)}_{\beta_1\dots\beta_{2s}}-\text{traces}\,. \tag{7.1}$$

In what follows we shall therefore focus on obtaining the field equations obeyed by the components of $\Phi^{(0;0)}$. Clearly, since Φ is a representation space of $hs(2,2|4)$, as given by (4.11), and the field content of Φ is in one-to-one correspondence with the massless spectrum listed in Table 3, the lowest energies E_0 of the components in $\Phi^{(0;0)}$ must be given by (1.5), as we shall verify explicitly using the field equations derived below. In deriving lowest energy labels E_0 from the various physical equations we use the harmonic analysis described in appendix B.

[9]This condition does not arise in the bosonic case [1], since $Y=0$ in that case.

7.1.1 The matter field equations ($s = 0, \frac{1}{2}$)

The scalar Klein-Gordon equations follow from the following components of the scalar master equation (6.7) [1]

$$\partial_\mu \Phi = \frac{1}{2}(\Gamma_\mu)^{\alpha\beta}\Phi^{(1;0)}_{\alpha,\beta}\,, \tag{7.2}$$

$$\nabla_\mu \Phi_{[\alpha,\beta]} = \frac{1}{2}(\Gamma_\mu)^{\gamma\delta}\left[\Phi^{(2;0)}_{[\alpha|\gamma,|\beta]\delta} + \Phi^{(2;1)}_{[\alpha|\gamma,|\beta]\delta} + \Phi^{(2;2)}_{[\alpha|\gamma,|\beta]\delta}\right] - \frac{1}{2}(\Gamma_\mu)_{\alpha\beta}\Phi\,, \tag{7.3}$$

where all components are coefficients in the expansion (5.1). The second equation decomposes as

$$\nabla_\mu \Phi^{(1;0)}_{\alpha,\beta} = \frac{1}{2}(\Gamma_\mu)_{\gamma\delta}\left[\Phi^{(2;0)}_{[\alpha|\gamma,|\beta]\delta} + \Phi^{(2;2)}_{[\alpha|\gamma,|\beta]\delta}\right] - \frac{1}{2}(\Gamma_\mu)_{\alpha\beta}\Phi\,, \tag{7.4}$$

$$\nabla_\mu \Phi^{(1;1)}_{\alpha,\beta} = \frac{1}{2}(\Gamma_\mu)_{\gamma\delta}\Phi^{(2;1)}_{[\alpha|\gamma,|\beta]\delta}\,. \tag{7.5}$$

From (5.11) and the expansion (5.12) we read off

$$\Phi^{(1;1)}_{\alpha,\beta} = \frac{Y}{4}C_{\alpha\beta}\Phi\,, \tag{7.6}$$

$$\Phi^{(2;1)}_{\alpha\gamma,\beta\delta} = \frac{2Y}{3}\Phi^{(1;0)}_{(\alpha|(\beta|}C_{|\gamma)|\delta)}\,, \tag{7.7}$$

$$\Phi^{(2;2)}_{\alpha\gamma,\beta\delta} = \left(\frac{2}{5} + \frac{Y^2}{10}\right) C_{(\alpha|(\beta|}C_{|\gamma)|\delta)}\Phi\,. \tag{7.8}$$

Taking the divergence of (7.2) and using (7.4) and (7.8) and the Fierz identity

$$(\Gamma^a)^{\alpha\beta}(\Gamma_a)^{\gamma\delta}\Phi^{(2;0)}_{\alpha\beta\gamma\delta} = 0\,, \tag{7.9}$$

we find

$$(\nabla^\mu\partial_\mu + 4 - \frac{Y^2}{4})\Phi = 0\,. \tag{7.10}$$

From this it follows that the lowest energy is $E_0 = 2 + \frac{1}{2}|Y|$, in accordance with (1.5). Thus, at level $\ell = 0$, the $20'_0$-plet, 10_2-plet and complex 1_4-plet have $E_0 = 2$, 3 and 4, respectively, and at level $\ell = 1$ the real 1_0-plet has $E_0 = 2$. The remaining equation (7.5) is an identity, upon the use of (7.6) and (7.7), and $\Phi^{(2;0)}_{\alpha\beta\gamma\delta}$ contains the non-vanishing second derivatives.

The spin $s = 1/2$ field equations follow from the following components of the scalar master equation (6.7):

$$\nabla_\mu \Phi_\alpha = \frac{1}{2}(\Gamma_\mu)^{\beta\gamma}\Phi_{\alpha\beta,\gamma} = \frac{1}{2}(\Gamma_\mu)^{\beta\gamma}\left[\Phi^{(1;0)}_{\alpha\beta,\gamma} + \frac{2Y}{5}\Phi_{(\alpha}C_{\beta)\gamma}\right], \tag{7.11}$$

where we have used (5.11). The Γ^μ-trace yields the Dirac equation

$$\left((\Gamma^\mu)_\alpha{}^\beta\nabla_\mu - \delta_\alpha^\beta\frac{Y}{2}\right)\Phi_\beta = 0\,. \tag{7.12}$$

This gives the lowest energy $E_0 = 2 + \frac{1}{2}|Y|$, in accordance with (1.5). Thus, at level $\ell = 0$ the 20_1-plet and the 4_3-plet have $E_0 = \frac{5}{2}$ and $\frac{7}{2}$, respectively, and at level $\ell = 1$ the 4_1-plet has $E_0 = \frac{5}{2}$. The non-vanishing derivatives are contained in $\Phi^{(1;0)}_{\alpha\beta,\gamma} = (\Gamma^a)_{\gamma(\alpha}\Psi_{a\,\beta)}$, where $\Psi_{a\,\alpha}$ is a Γ-traceless vector-spinor.

7.1.2 The self-duality equations ($s \geq 1$)

The part of (6.7) that is totally symmetric in α and β indices can be written as

$$\nabla_\mu \Phi^{(0;0)}_{\nu\rho\,\alpha_1\ldots\alpha_{2s-2}} = \frac{1}{2}(\Gamma_{\nu\rho})^{\beta\gamma}(\Gamma_\mu)^{\delta\epsilon}\left[\Phi^{(1;0)}_{\beta\gamma\delta\epsilon\alpha_1\ldots\alpha_{2s-2}} + \Phi^{(1;1)}_{\beta\gamma\delta\epsilon\alpha_1\ldots\alpha_{2s-2}}\right], \qquad (7.13)$$

where we use the notation of (5.1) and we have defined

$$\Phi^{(0;0)}_{\mu\nu\,\alpha_1\ldots\alpha_{2s-2}} = (\Gamma_{\mu\nu})^{\beta\gamma}\Phi^{(0;0)}_{\beta\gamma\alpha_1\ldots\alpha_{2s-2}}\,. \qquad (7.14)$$

From this definition and the membrane identity (6.14), it follows that

$$(\Gamma^\mu)_\beta{}^\gamma\Phi^{(0;0)}_{\mu\nu\,\gamma\alpha_1\ldots\alpha_{2s-2}} = 0\,. \qquad (7.15)$$

Using this result, we can invert (7.14) to obtain

$$\Phi^{(0;0)}_{\beta\gamma\alpha_1\ldots\alpha_{2s-2}} = \frac{1}{8}(\Gamma^{\mu\nu})_{\beta\gamma}\Phi^{(0;0)}_{\mu\nu\,\alpha_1\ldots\alpha_{2s-2}}\,. \qquad (7.16)$$

The total symmetry of the left hand side of (7.16) is ensured by the identity (7.15). Substituting the trace part $\Phi^{(1;1)}$ in (7.13) by the expression (5.11) gives

$$\begin{aligned}\nabla_\mu \Phi^{(0;0)}_{\nu\rho\,\alpha_1\ldots\alpha_{2s-2}} &= \frac{1}{2}(\Gamma_{\nu\rho})^{\beta\gamma}(\Gamma_\mu)^{\delta\epsilon}\Phi^{(1;0)}_{\beta\gamma\delta\epsilon\alpha_1\ldots\alpha_{2s-2}} + \\ &\quad + \frac{Y}{2(s+2)}\Big[(\Gamma_{\mu\nu\rho})^{\beta\gamma}\Phi^{(0;0)}_{\beta\gamma\alpha_1\ldots\alpha_{2s-2}} + \\ &\qquad + (s-1)(\Gamma_\mu)_{(\alpha_1}{}^\beta\Phi^{(0;0)}_{\nu\rho\,\alpha_2\ldots\alpha_{2s-2})\beta}\Big]\,. \end{aligned} \qquad (7.17)$$

In computing the curl and divergence of $\Phi_{\mu\nu\,\alpha_1\ldots\alpha_{2s-2}}$ the contributions from $\Phi^{(1;0)}$ vanish due to the Fierz identities (6.10) and (6.14). It follows that:[10]

$$\nabla_{[\mu}\Phi^{(0;0)}_{\nu\rho]\,\alpha_1\ldots\alpha_{2s-2}} = -\frac{iY}{12}\epsilon_{\mu\nu\rho}{}^{ab}\Phi^{(0;0)}_{ab\,\alpha_1\ldots\alpha_{2s-2}}\,, \qquad (7.18)$$

$$\nabla^\mu\Phi^{(0;0)}_{\mu\nu\,\alpha_1\ldots\alpha_{2s-2}} = 0\,, \qquad (7.19)$$

where we have used the Fierz identity (6.13) in obtaining (7.18). For $Y \neq 0$ the divergence equation (7.19) follows from the curl equation (7.18). Taking the divergence of (7.18) and using (7.19) gives (for all Y)

$$\left(\nabla^\rho\nabla_\rho + 2s + 4 - \frac{Y^2}{4}\right)\Phi^{(0;0)}_{\mu\nu\,\alpha_1\ldots\alpha_{2s-2}} = 0\,. \qquad (7.20)$$

[10] Our conventions are $[\nabla_\mu, \nabla_\nu]\psi_{a\,\alpha} = R_{\mu\nu,a}{}^b\psi_{b\,\alpha} - \frac{1}{2}(\Gamma_{\mu\nu})_\alpha{}^\beta\psi_{a\,\beta}$ and $\epsilon^{abcde} = i\Gamma^{abcde}$.

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We find (using $2(j_L - j_R) = \Delta = -Y$ in (B.2)) that the lowest energy of $\Phi^{(0;0)}_{\mu\nu\,\alpha_1\ldots\alpha_{2s-2}}$ is given by $E_0 = s+2$ in accordance with (1.5).

The equations (7.18) and (7.19) describe:

- The physical field equations for the underlined states shown in Table 6 (all of which have $|Y| > 1$). These are only realized as two-form potentials in the scalar master field, i.e. they have no duals in the master gauge field. In particular, at level $\ell = 0$ we find the self-duality equation for the 6-plet of two-form potentials arising in the supergravity theory.

- The physical field equations for the remaining spin $s \geq 2$ states in table 6 (all of which have $|Y| > 1$). These are realized as two-form potentials in the scalar master field, and their field equations take the form of generalized higher spin self-duality equations in five dimensions. As we shall see below, these two-form potentials have dual potentials in the master gauge field.

- The consistency equations satisfied by the spin $s \geq 1$ Weyl tensors listed in table 4 (all of which have $|Y| \leq 1$). These are non-vanishing curvatures of physical potentials in the master gauge field.

7.2 The master curvature constraint

For $|\Delta| \leq 1$ the set of spin s gauge fields and their curvature constraints are equivalent to the those used in [14, 15] to describe a physical spin s field. Those for $|\Delta| > 1$ are of new type. As we shall see, they are dualized into the physical two-form potentials with $|Y| > 1$ found in Φ in the previous section.

The Yang-Mills equations ($s = 1$, $\Delta = 0$)

For spin $s = 1$ the curvature constraint (6.1) takes the form $F_{\mu\nu} = \frac{1}{8}\Phi^{(0;0)}_{\mu\nu}$. Identifying $\Phi^{(0;0)}_{\mu\nu}$ with the field strength for the spin $s = 1$ gauge fields, i.e. the 15-plet at level $\ell = 0$ and the $(15+1)$-plet at level $\ell = 1$, the linearized Yang-Mills equations follow from (7.19), while (7.18) becomes the Bianchi identity (as $Y = 0$).

The Field equations for $s \geq \frac{3}{2}$, $\Delta = 0, \pm 1$

For $|\Delta| \leq 1$ the curvature constraint (6.1) decomposes into physical field equations, generalized torsion equations (except for spin $s = \frac{3}{2}$, since there are no auxiliary spin $s = \frac{3}{2}$ gauge fields), and identities that give the components of the curvature which are non-vanishing on-shell, which by definition are the generalized Weyl tensors.

For $\Delta = 0$ the physical gauge fields are [1, 14]

$$\begin{array}{l}\Delta = 0\\ s = 2,3,\ldots\end{array} \quad : \quad A^{(s-1;0)}_{\mu\,\alpha_1\ldots\alpha_{s-1},\beta_1\ldots\beta_{s-1}} = (\Gamma^{a_1})_{\alpha_1\beta_1}\cdots(\Gamma^{a_{s-1}})_{\alpha_{s-1}\beta_{s-1}} A_{\mu,a_1\ldots a_{s-1}}\,, \quad (7.21)$$

where $a_1 \dots a_{s-1}$ are symmetric and traceless, which implies that $A_{(a_1,a_2\dots a_s)}$ is symmetric and doubly traceless. The SU(4) content is given in the $\Delta = 0$ sector of Table 4. The $(s-1;0)$ component of (6.1) is a generalized torsion equation which yields the generalized spin s Lorentz connection $A^{(s-2;0)}_{\mu\,\alpha_1\dots\alpha_{s-1},\beta_1\dots\beta_{s-1}}$ in terms of one derivative of the physical gauge field. The $(s-2;0)$ component of (6.1) contains the physical field equation, which is of second order and describe a massless spin s field with lowest energy $E_0 = s+2$. It also contains components from which one can solve for the auxiliary spin s gauge field $A^{(s-3;0)}$. The $(s-3;0)$, ..., $(1;0)$ components of (6.1) yield the auxiliary spin s gauge fields $A^{(s-4;0)}$, ..., $A^{(0;0)}$, respectively, in terms of derivatives of the physical gauge field. Finally, the $(0;0)$ component of (6.1) sets the non-vanishing component of the spin s curvature equal to $\Phi^{(0;0)}_{\alpha_1\dots\alpha_{2s}}$. This generalized spin s Weyl tensor is thus given by s derivatives of the physical gauge field, and obeys the equations (7.18) and (7.19) derived above for $Y = 0$.

For $|\Delta| = 1$ the physical gauge fields are [1, 15, 18]

$$\begin{array}{l}\Delta = 1\\ s=\frac{3}{2},\frac{5}{2},\dots\end{array} \quad : \quad A^{(s-\frac{3}{2};0)}_{\mu\,\alpha_1\dots\alpha_{s-\frac{3}{2}}\gamma,\beta_1\dots\beta_{s-\frac{3}{2}}} = (\Gamma^{a_1})_{\alpha_1\beta_1}\cdots(\Gamma^{a_{s-\frac{3}{2}}})_{\alpha_{s-\frac{3}{2}}\beta_{s-\frac{3}{2}}}\psi_{\mu,a_1\dots a_{s-\frac{3}{2}}\gamma}\,, \tag{7.22}$$

where $a_1 \dots a_{s-\frac{3}{2}}$ are symmetric and $\psi_{\mu,a_1\dots a_{s-\frac{3}{2}}\gamma}$ is Γ-traceless ($s \geq \frac{5}{2}$):

$$(\Gamma^b)_\gamma{}^\delta \psi_{\mu,ba_1\dots a_{s-\frac{5}{2}}\delta} = 0\,.$$

The Γ-tracelessness implies SO(4,1)-tracelessness ($s \geq \frac{7}{2}$). The (complex) dimension of the tangent space irrep carried by (7.22) is $\frac{2}{3}(s+\frac{3}{2})(s+\frac{1}{2})(s-\frac{1}{2})$. The SU(4) content is given in Table 4 for $|\Delta| = 1$. The $(s-\frac{3}{2};0)$ component of (6.1) contains the first order field equation, which gives $E_0 = s+2$. For $s \geq \frac{5}{2}$ it also yields the auxiliary spin s gauge field $A^{(s-\frac{5}{2};0)}_{\mu\,\alpha_1\dots\alpha_{s-\frac{3}{2}}\gamma,\beta_1\dots\beta_{s-\frac{3}{2}}}$ in terms of one derivative of the physical gauge field. The remaining components of (6.1) yield the auxiliary spin s gauge fields $A^{(s-\frac{7}{2};0)}$, ..., $A^{(0;0)}$ and the generalized Weyl tensor $\Phi^{(0;0)}_{\alpha_1\dots\alpha_{2s}}$ in terms of derivatives of the physical gauge field. The Weyl tensor satisfies (7.18) and (7.19) for $Y = -\Delta = \pm 1$.

To summarize, the gauge fields with $|\Delta \leq 1$, where Δ is given by (2.17), contain bosonic gauge fields with spin $s = 1, 2, \dots$ that have tangent space multi-spinor indices in one-to-one correspondence [1, 10] with two-row SO(4,1) Young tableaux with first row containing $s-1$ boxes and the second row containing t boxes where $0 \leq t \leq s-1$, and fermionic gauge fields with spin $s = \frac{3}{2}, \frac{5}{2}, \dots$ corresponding to tensor-spinors with SO(4,1) indices in the Young tableaux with first row containing $s-\frac{3}{2}$ boxes and second row containing t boxes with $0 \leq t \leq s-\frac{3}{2}$. This set admits well-known curvature constraints in the tensor basis [14, 15]. In this paper we have cast them into the multi-spinor basis (this was first done in the bosonic case in [1]). For $s \geq \frac{3}{2}$ the curvature constraints give rise to physical field equations. For $s = 1$

the (linearized) Yang-Mills equation follows from the master scalar constraint. The physical field equations are obeyed by the subset of the gauge fields with $t = 0$, given in (7.21) and (7.22). The remaining ones ($s \geq 2$) are auxiliary.

The equations for $|\Delta| > 1$, $s \geq 2$ and their dualization

Let us begin by examining the case of $s = 2$ and $|\Delta| = 2$. The only gauge field present is $A_{\mu\,\alpha\beta}$ (the 16_2-plet listed in Table 5). The curvature constraint (6.6) for $A_{\mu\,\alpha\beta}$ reads:

$$F_{\mu\nu\,\alpha\beta} := 2\nabla_{[\mu}A_{\nu]\,\alpha\beta} + 2(\Gamma_{[\mu})_{(\alpha}{}^{\epsilon}A_{\nu]\,\beta)\epsilon} = \frac{1}{8}\Phi^{(0;0)}_{\mu\nu\,\alpha\beta}\,, \tag{7.23}$$

where $\Phi^{(0;0)}_{\mu\nu\,\alpha\beta}$ is the physical two-form potential obeying the self-duality equations (7.18). Using (7.15) and imposing the gauge condition

$$(\Gamma^\mu)_\alpha{}^\gamma A_{\mu\,\gamma\beta} = 0\,,$$

we find that $A_{\mu\,\alpha\beta}$ satisfies the linear field equation

$$(\Gamma^\nu)_\alpha{}^\gamma \nabla_\nu A_{\mu\,\gamma\beta} + 2A_{\mu\,\alpha\beta} = 0\,. \tag{7.24}$$

Differentiating this[11] one finds the lowest energy $E_0 = 4$. Thus, interestingly enough, at the linearized level the spin $s = 2$ and $|\Delta| = 2$ state with $(j_L, j_R) = (\frac{3}{2}, \frac{1}{2})$ has dual formulations in terms of either a one-form or a two-form potential. An analogous result is valid for $s = \frac{5}{2}$ and $|\Delta| = 3$, where the physical gauge field $A_{\mu\,\alpha\beta\gamma}$ obeys a first order equation with energy $E_0 = \frac{9}{2}$. For $s = 3$ there is no gauge field with $|\Delta| = 4$ (which would have been physical at the linearized level). For $s = 3$ and $|\Delta| = 2$ the gauge fields are $A^{(0;0)}_{\mu\,\alpha\beta\gamma,\delta}$ and $A^{(1;0)}_{\mu\,\alpha\beta\gamma,\delta}$ (the level $\ell = 1$ and $\ell = 2$ 6_2-plets listed in in table 5). Their curvature constraint is given by

$$F_{\mu\nu\,\alpha\beta\gamma,\delta} := 2\nabla_{[\mu}A_{\nu]\,\alpha\beta\gamma,\delta} + 3(\Gamma_{[\mu})_{(\alpha}{}^{\epsilon}A_{\nu]\,\beta\gamma)\epsilon,\delta} - (\Gamma_{[\mu})_\delta{}^\epsilon A_{\nu]\,\alpha\beta\gamma,\epsilon} = \frac{1}{8}\Phi^{(0;0)}_{\mu\nu\,\alpha\beta\gamma\delta}\,. \tag{7.25}$$

This constraint decomposes into $(1;0)$, $(1;1)$ and $(0;0)$ parts. The $(1;1)$ part is obeyed identically. The $(1;0)$ part can be used to solve for $A^{(0;0)}$ in terms of $A^{(1;0)}$:

$$A^{(0;0)}_{\mu\,\alpha\beta\gamma,\delta} = \frac{1}{16}(\Gamma^{ab})_{(\alpha\beta}\left(2\Omega_{\mu a,b} - \Omega_{ab,\mu}\right)_{\gamma\delta)}\,, \tag{7.26}$$

[11] More generally, a curvature constraint of the form

$$F_{\mu\nu\,\alpha_1\ldots\alpha_m} := 2\nabla_{[\mu}A_{\nu]\,\alpha_1\ldots\alpha_m} + m(\Gamma_{[\mu})_{(\alpha_1}{}^{\beta}A_{\nu]\,\alpha_2\ldots\alpha_m)\beta} = \frac{1}{8}\Phi^{(0;0)}_{\mu\nu\,\alpha_1\ldots\alpha_m}$$

leads to the first order equation

$$(\Gamma^\nu)_\beta{}^\gamma\nabla_\nu A_{\mu\,\gamma\alpha_1\ldots\alpha_{m-1}} + \frac{m+2}{2}A_{\mu\,\beta\alpha_1\ldots\alpha_{m-1}} = 0\,,$$

in the gauge $\Gamma^\mu A_\mu = 0$. Differentiating this using $(\not\nabla)^2 A_\mu = (\nabla^2 + 5 + m)A_\mu$ one finds the lowest energy $E_0 = \frac{1}{2}m + 3 = s + 2$. The above constraint arises for $m = 2, 3$ in gauging $hs(2,2|4)$ (due to the condition (2.5) there is no gauge field with $|\Delta| = 4$ and spin $s = 3$). Cases with $m > 3$ are expected to arise in gauging enlarged versions of $hs(2,2|4)$.

where

$$\Omega_{ab,c\,\alpha\beta} = 2(\Gamma_c)^{\gamma\delta}\left(\nabla_{[a}A^{(1;0)}_{b]\,\alpha\beta\gamma,\delta} + (\Gamma_{[a})_{(\alpha}{}^{\epsilon}A^{(1;0)}_{b]\beta)\epsilon\gamma,\delta}\right). \tag{7.27}$$

Finally, the $(0;0)$ part yields the curvature of $A^{(0;0)}$ in terms of $\Phi^{(0;0)}_{\mu\nu\,\alpha\beta\gamma\delta}$. Thus, one can solve locally for $A^{(0;0)}$ and $\Phi^{(0;0)}$ in terms of derivatives of $A^{(1;0)}$ *without* going on-shell. By taking the Γ-trace of the $(0;0)$ component of (7.25) and using (7.15), one obtains a second order field equation for $A^{(1;0)}$. This equation involves rotations of spinor indices, however, which implies that it is effectively a higher derivative equation. Thus the physical spin $s=3$ fields with $|Y|=2$ obeying proper physical field equations are the two-form potentials found in Φ. The curvature constraint should therefore be interpreted as a duality relation between the two-form and one-form fields (rather than a field equation for the latter).

The analysis of the spin $s=3$ and $|\Delta|=2$ cases generalizes to $s>3$ and $|\Delta|>1$. Thus the curvature constraint (6.1) (except its $(0;0)$ part) can be used to solve for auxiliary gauge fields with $|\Delta|>1$ in terms of derivatives of independent gauge fields without going on-shell. The independent gauge fields are $A^{(s-2;0)}$ $(s=2,3,\ldots)$ and $A^{(s-\frac{3}{2};0)}$ $(s=\frac{5}{2},\frac{7}{2},\ldots)$. The remaining $(0;0)$ part of the constraint, which involves $s-1$ derivatives of $A^{(s-2;0)}$ $(s=2,3,\ldots)$ or $s-\frac{1}{2}$ derivatives of $A^{(s-\frac{3}{2};0)}$ $(s=\frac{5}{2},\frac{7}{2},\ldots)$, dualizes the independent gauge fields to corresponding higher spin two-form potentials in $\Phi^{(0;0)}$.

Thus, the independent gauge fields in the $|\Delta|>1$ sector are:

$$\begin{array}{l}\Delta=2\\ s=2,3,\ldots\end{array} : A^{(s-2;0)}_{\mu\,\alpha_1\ldots\alpha_s,\beta_1\ldots\beta_{s-2}} = (\Gamma^{a_1})_{\alpha_1\beta_1}\cdots(\Gamma^{a_{s-2}})_{\alpha_{s-2}\beta_{s-2}}(\Gamma^{bc})_{\alpha_{s-1}\alpha_s}\times$$
$$\times A_{\mu,bc,a_1\ldots a_{s-2}}\,, \tag{7.28}$$

$$\begin{array}{l}\Delta=3\\ s=\frac{5}{2},\frac{7}{2},\ldots\end{array} : A^{(s-\frac{5}{2};0)}_{\mu\,\alpha_1\ldots\alpha_{s-\frac{1}{2}}\gamma,\beta_1\ldots\beta_{s-\frac{5}{2}}} = (\Gamma^{a_1})_{\alpha_1\beta_1}\cdots(\Gamma^{a_{s-\frac{5}{2}}})_{\alpha_{s-\frac{5}{2}}\beta_{s-\frac{5}{2}}}(\Gamma^{bc})_{\alpha_{s-\frac{3}{2}}\alpha_{s-\frac{1}{2}}}\times$$
$$\times\,\psi_{\mu,bc,a_1\ldots a_{s-\frac{5}{2}}\gamma}\,, \tag{7.29}$$

$$\begin{array}{l}\Delta=4\\ s=4,5,\ldots\end{array} : A^{(s-3;0)}_{\mu\,\alpha_1\ldots\alpha_{s+1},\beta_1\ldots\beta_{s-3}} = (\Gamma^{a_1})_{\alpha_1\beta_1}\cdots(\Gamma^{a_{s-3}})_{\alpha_{s-3}\beta_{s-3}}(\Gamma^{b_1c_1})_{\alpha_{s-2}\alpha_{s-1}}\cdot$$
$$\cdot\,(\Gamma^{b_2c_2})_{\alpha_s\alpha_{s+1}}A_{\mu,b_1c_1,b_2c_2,a_1\ldots a_{s-3}}\,. \tag{7.30}$$

Here the Young tableaux for $bc,a_1\ldots a_{s-2}$ $(|\Delta|=2)$ has $s-1$ boxes in the first row and 1 box in the second row and dimension $(s+2)(s+\frac{1}{2})(s-1)$; the Young tableaux for $b_1c_1,b_2c_2,a_1\ldots a_{s-3}$ $(|\Delta|=4)$ has $s-1$ boxes in the first row and 2 boxes in the second row and dimension $\frac{5}{3}(s+3)(s+\frac{1}{2})(s-2)$. The tensor-spinor in the right hand side of (7.29), which has (complex) dimension $\frac{4}{3}(s+\frac{5}{2})(s+\frac{1}{2})(s-\frac{3}{2})$ obeys suitable Γ-trace conditions. All these gauge fields are dualized to two-form potentials, which obey the physical field equation (7.18) realizing the $|Y| > 1$, $s \geq 2$ sector of the spectrum in Table 3. The spin $s = 2$, $\Delta = 2$ gauge field $A_{\mu\,\alpha\beta}$ and spin $s = \frac{5}{2}$, $\Delta = 3$ gauge field $A_{\mu\,\alpha\beta\gamma}$ also obey physical field equations, which assume the form (7.24) in a fixed gauge.

To summarize, the gauge fields with $|\Delta| > 1$) are of new type. All gauge fields in this set have spin $s \geq 2$. For $s = 2, 3, \ldots$ and $|\Delta| = 2, 4$ their tangent space multi-spinor indices are in one-to-one correspondence with two-row SO(4,1) Young tableaux with first row containing $s-1$ boxes and the second row containing t boxes where $\frac{1}{2}|\Delta| \leq t \leq s-1$. For $s = \frac{5}{2}, \frac{7}{2}, \ldots$ and $\Delta| = 3$ the multi-spinor indices are equivalent to tensor-spinors whose SO(4,1) Young tableaux have $s - \frac{3}{2}$ boxes in the first row and t boxes in the second row with $1 \leq t \leq s - \frac{3}{2}$. We have identified the linearized curvature constraints for the gauge fields in this set. As a result the gauge fields listed in (7.28-7.30), all of which have mixed symmetries, remain independent, while the other gauge fields can be expressed in terms of derivatives of the independent gauge fields without imposing any field equations. Higher derivative field equations for the independent gauge fields (for $s > 2$) arises by Γ-tracing the remaining curvature component. The Γ-traceless part of this component implies a dualization of the independent one-forms into (higher spin) two-form potentials contained in the master scalar field. These two-forms obey physical field equations generalizing the self-duality equation in five dimensions satisfied by the two-form potential in the gauged supergravity sector [11, 12].

8. Conclusions

In this paper we have taken the first step towards the construction of the full interacting theory by identifying the symmetry group, the full spectrum, the master fields required to describe the theory and the correct linearized equations of motion. In particular, we have shown how the linearized field equations of $D = 5, N = 8$ gauged supergravity are embedded in the theory. For example, the correct AdS_5 mass splitting among the 42 scalars and the self duality equation for the 6-plet of two-form potentials are obtained. These results provide nontrivial checks on some crucial aspects of the theory, such as the conditions imposed on the master scalar field, as well as the basic hypothesis that the full spectrum of the theory consists of all states resulting from the symmetric product of two $d = 4, N = 4$ Yang-Mills superdoubleton. One curious result is that the full spectrum exhibited in table 3 coincides precisely with the spectrum of higher spin $D = 4, N = 8$ supergravity [20] for all levels above the lowest one! Thus, the PSU(2,2|4) supermultiplets in levels $\ell = 1, 2, \ldots, \infty$ of the five-dimensional theory listed in table 3 seem to be also the supermultiplets of the AdS_4 superalgebra $OSp(8|4)$. The full spectra seem to differ only at level $\ell = 0$, where the distinct supergravity multiplets reside.

It is clear that the next step is to construct the full interacting theory. The existence of certain cubic interactions for the $|\Delta \leq 1$ gauge fields [10] indicates that there exist consistent interactions to all orders, though these necessarily require the inclusion of the "matter" fields contained in Φ. Experience from four spacetime dimensions, where the interacting full theory of massless higher spins exists, suggests that a natural framework for discussing the interactions is an extended spacetime with extra non-commutative "z"-space coordinates [19] (See [20] for a detailed study of the case of higher spin $D = 4, N = 8$ supergravity). In fact, the introduction of such a space is straightforward in the present case as well. Moreover, we expect that the full interacting higher spin $D = 5, N = 8$ supergravity equations will be described by z-extended and suitably "twisted" versions of our constraint equations (6.1) and (6.2), to be imposed, of course, on the full curvature two-form and full covariant derivative of the master zero-form. We also expect the resulting theory to yield a curvature expansion, just as in the four dimensional case. It would be interesting to compare the results of this expansion with the bulk predictions of the boundary CFT based on higher spin currents formed out of the $d = 4, N = 4$ Yang-Mills theory in the limit of zero t'Hooft coupling and large dimension of the gauge group.

Another direction, motivated by the question of whether there exists non-trivial interactions for tensionless opens strings, is to examine various extensions [13] of the higher spin gauge group $hs(2, 2|4)$, and their relation to (higher spin) super-doubletons with non-vanishing central charge. For example, one may restrict the τ_η-condition only to certain values of η. We expect that the new generators will give rise to gauge fields that are dual to additional higher spin two-form potentials. The extensions appears to prevent modding out the central charge, however, in which case the spectrum will contain infinitely many massless higher spin fields of each given spin. Whether such a degeneracy is natural from the point of view of string theory is not altogether obvious because the zero tension limit of type-IIB string theory is intrinsically nonperturbative from the worldsheet point of view (as $\alpha' \to \infty$) and not much is known about the space of soliton solutions in this setup that can be interpreted as physical states in AdS_5.[12]

An important obstacle in making progress towards establishing a duality between the free limit of $N = 4$ SYM and a higher spin gauge theory in the bulk is really our lack of knowledge about the quantization of type-IIB strings in $AdS_5 \times S^5$. Hardly anything is known about the string states beyond the low energy supergravity and the Kaluza-Klein modes in this case. It is true that some scattered and interesting variety of soliton solutions in type-IIB theory are known but their relevance or fate in the tensionless limit is not clear.

With this state of affairs, one may hope that the quantization of tensionless type-IIB string on $AdS_5 \times S^5$, unlike in the case of flat target, is more amenable to study.

[12] We thank M. Douglas for illuminating discussions on this point.

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If this approach proves to be just as difficult as the finite tension case, one avenue which still remains open and technically within reach, is to study the $4d$ CFT based on the free limit of $N = 4$ SYM (or its higher spin extension thereof) in detail, e.g. its correlation functions [3, 4], and to try to obtain information about the bulk theory they point to. In this way, one may not only try to establish a connection with an interacting higher spin gauge theory in the bulk, but by establishing the properties of such a theory (e.g. its spectrum, symmetries and symmetry breaking mechanisms) one may also get an idea about where to look for the states of the tensionless type IIB string.

Acknowledgments

This research project has been supported in part by NSF Grant PHY-0070964. The work of P.S. is part of the research program of Stichting voor Fundamenteel Onderzoek der Materie (Stichting FOM). We thank Feza Gürsey Institute, Groningen University and Benasque Center for Science for hospitality. We also thank O. Aharony, M. Berkooz, M. Douglas, A. Polyakov, K. Skenderis and A. Zaffaroni for useful discussions on various aspects of tensionless strings.

A. Decomposition of spectrum into massless $\mathrm{PSU}(2,2|4)$ irreps

The spectrum $\mathcal{S}$ of the $hs(2,2|4)$ gauge theory is given by the symmetric part of the tensor product of two Maxwell multiplets (superdoubletons with $Z = 0$). It is well-known [6] that this space decomposes into massless irreps of $\mathrm{PSU}(2,2|4)$. In order to apply this result to the construction of the $hs(2,2|4)$ gauge theory we need to find by what multiplicity each massless $\mathrm{PSU}(2,2|4)$ multiplet occurs in $\mathcal{S}$.

To this end, we split the $\mathrm{SU}(2,2)\times\mathrm{SU}(4)$-covariant super-oscillator $Z_R \equiv (y_{\underline{\alpha}}, \theta_i)$, $\underline{\alpha} = 1,\ldots,4$; $i = 1,\ldots,4$, into $\mathrm{SU}(2)_L \times \mathrm{SU}(2)'_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)'_R$-covariant super-oscillators ξ_A and η^K as (see (A.8) and (A.8)):

$$\begin{aligned} Z_R &\to \sqrt{2}(\xi_A, \eta^K)\,, \\ \xi_A &= (\xi_a, \xi_\alpha)\,,\ a = 1,2;\ \alpha = 1,2\,, \\ \eta^K &= (\eta^k, \eta^\kappa)\,,\ k = 1,2;\ \kappa = 1,2\,, \end{aligned} \tag{A.1}$$

where $(-1)^a = -(-1)^\alpha = (-1)^k = -(-1)^\kappa = 1$. Defining

$$\xi^A = (\xi_A)^\dagger\,, \quad \eta_K = (\eta^K)^\dagger\,, \tag{A.2}$$

the oscillator algebra (1.1) is equivalent to the following graded commutation rules:

$$\xi_A \star \xi^B - (-1)^{AB}\xi^B \star \xi_A = \delta_A^B\,, \quad \eta_K \star \eta^L - (-1)^{KL}\eta^L \star \eta_K = \delta_K^L\,. \tag{A.3}$$

The vacuum state $|0\rangle$ of the oscillator Fock space is defined by

$$\xi_A|0\rangle = 0\,, \qquad \eta_K|0\rangle = 0\,. \tag{A.4}$$

The $\mathrm{SU}(2,2|4)\times\mathrm{U}(1)_Y$ generators decompose into

$$\Lambda_{AK} := \xi_A\eta_K\,, \quad \xi^A\xi_B\,, \qquad \eta^K\eta_L \tag{A.5}$$

and hermitian conjugates. The energy operator is given by

$$E = \frac{1}{2}(\xi^a\xi_a + \eta^k\eta_k + 2)\,, \tag{A.6}$$

and the U(1)-charges are given by

$$Z = \frac{1}{2}(\xi^a\xi_a + \xi^\alpha\xi_\alpha - \eta^k\eta_k - \eta^\kappa\eta_\kappa)\,, \qquad Y = \xi^\alpha\xi_\alpha - \eta^\kappa\eta_\kappa\,. \tag{A.7}$$

The lowest weight states of the maximal bosonic subalgebra $B = \mathrm{SU}(2,2)\times\mathrm{SU}(4)\times\mathrm{U}(1)_Z \subset \mathrm{SU}(2,2|4)$ are annihilated by the bosonic energy-lowering operators Λ_{ak} and $\Lambda_{\alpha\kappa}$ and labeled by the energy E_0, the $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ spins (j_L, j_R), an SU(4) irrep R and a central $\mathrm{U}(1)_Z$ charge Z. They also carry a $\mathrm{U}(1)_Y$ charge Y.

The split (A.1) defines a realization of the maximal compact supergroup $\mathrm{SU}(2|2)_L\times\mathrm{SU}(2|2)_R \subset \mathrm{SU}(2,2|4)$. The bosonic subalgebra $B' = \mathrm{SU}(2)_L\times\mathrm{SU}(2)'_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(2)'_R \subset \mathrm{SU}(2|2)_L\times\mathrm{SU}(2|2)_R$ has the generators:

$$\begin{aligned} L^a_b &= \xi^a\xi_b - \frac{1}{2}\delta^a_b\xi^c\xi_c\,, & L'^\alpha_\beta &= \xi^\alpha\xi_\beta - \frac{1}{2}\delta^\alpha_\beta\xi^\gamma\xi_\gamma\,, \\ R^k_l &= \eta^k\eta_l - \frac{1}{2}\delta^k_l\eta^m\eta_m\,, & R'^\kappa_\lambda &= \eta^\kappa\eta_\lambda - \frac{1}{2}\delta^\kappa_\lambda\eta^\mu\eta_\mu\,. \end{aligned} \tag{A.8}$$

The Clifford vacuum, $|\Omega\rangle$, of an SU(2,2) supermultiplet is a lowest weight state annihilated by bosonic as well as fermionic energy-lowering operators, that is

$$\Lambda_{AK}|\Omega\rangle = 0\,. \tag{A.9}$$

This ground state consists of a set of lowest weight states of the bosonic subalgebra B forming a supermultiplet of $\mathrm{SU}(2|2)_L\times\mathrm{SU}(2|2)_R$. The states in $|\Omega\rangle$ can therefore alternatively be labeled by their B' highest weights $(j_L, j_R; j'_L, j'_R)$ and $\mathrm{U}(1)_Y$ charge. Parametrizing such a state with central charge $Z = \frac{1}{2}(m-n)$ as

$$|\Omega\rangle = \psi_{A_1\ldots A_m;K_1\ldots K_n}\xi^{A_1}\cdots\xi^{A_m}\eta^{K_1}\cdots\eta^{K_n}|0\rangle\,, \tag{A.10}$$

and imposing (A.9) one finds $mn\psi_{AA_1\ldots A_{m-1};KK_1\ldots K_{n-1}} = 0$. Thus the Fock space of the superoscillators decomposes into superdoubletons with with Clifford vacua $|\Omega\rangle = \xi^{A_1}\cdots\xi^{A_{2j}}|0\rangle$ and $|\Omega\rangle = \eta^{K_1}\cdots\eta^{A_{2j}}|0\rangle$, $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$, with central charge $Z = \pm j$, respectively. Acting on them with the remaining supercharges $\xi^\alpha\eta^k$ and $\xi^a\eta^\kappa$ gives the supermultiplets [6] listed in tables 1 and 2.

In order to decompose the product of two doubletons we describe the tensor product by introducing two flavors of oscillators labeled by an index $r = 1, 2$ as follows:

$$[\xi(r)_A, \xi^B(s)] = \delta_{rs}\delta^B_A\,, \qquad [\eta(r)_K, \eta^L(s)] = \delta_{rs}\delta^L_K\,, \qquad r, s = 1, 2\,. \tag{A.11}$$

An $\mathrm{SU}(2, 2|4) \times \mathrm{U}(1)_Y$ generator, Λ say, is represented by $\Lambda = \sum_{r=1,2} \Lambda(r)$. In particular

$$E = 2 + \frac{1}{2}\sum_{r=1,2} (\xi^a(r)\xi_a(r) + \eta^k(r)\eta_k(r))\,.$$

We are interested in the product $\mathcal{D} \otimes \mathcal{D}$ of two $Z = 0$ weight spaces $\mathcal{D}$, each of which consist of the states:

$$\mathcal{D} = |0\rangle \oplus \left(\xi^A\eta^K|0\rangle\right) \oplus \left(\xi^A\xi^B\eta^K\eta^L\right)|0\rangle \oplus \ldots \tag{A.12}$$

Parametrizing a ground state in $\mathcal{D} \otimes \mathcal{D}$ as ($n \in \mathsf{Z}$):

$$\begin{aligned}|\Omega\rangle = &\sum_{m=0}^{n} \psi^{(m)}_{A_1\ldots A_m,B_1\ldots B_{n-m};K_1\ldots K_m,L_1\ldots L_{n-m}} \xi^{A_1}(1)\cdots\xi^{A_m}(1)\xi^{B_1}(2)\cdots \\ &\cdots\xi^{B_{n-m}}(2)\eta^{K_1}(1)\cdots\eta^{K_m}(1)\eta^{L_1}(2)\cdots\eta^{L_{n-m}}(2)|0\rangle\,.\end{aligned} \tag{A.13}$$

and imposing $(\Lambda_{CM}(1) + \Lambda_{CM}(2))|\Omega\rangle = 0$ we find the conditions

$$\begin{aligned}\psi^{(1)}_{C,B_1\cdots B_{n-1};M,L_1\cdots L_{n-1}} + n^2\psi^{(0)}_{CB_1\cdots B_{n-1};ML_1\cdots L_{n-1}} &= 0\,,\\ 4\psi^{(2)}_{CA_1,B_1\cdots B_{n-2};MK_1,L_1\cdots L_{n-2}} + (-1)^{CA_1+MK_1}(n-1)^2\psi^{(1)}_{A_1,CB_1\cdots B_{n-2};K_1,ML_1\cdots L_{n-2}} &= 0\,,\\ &\vdots\\ n^2\psi^{(n)}_{CA_1\cdots A_{n-1};MK_1\cdots K_{n-1}} + \\ (-1)^{C(A_1+\cdots+A_{n-1})+M(K_1+\cdots+K_{n-1})}\psi^{(n-1)}_{A_1\cdots A_{n-1},C;K_1\cdots K_{n-1},M} &= 0\,.\end{aligned} \tag{A.14}$$

It follows that for each n there is a unique ground state. We find that the tensor product $\mathcal{D} \otimes \mathcal{D}$ decomposes into massless supermultiplets $\mathcal{D}_{s_{\max}}$ (with maximal spin $s_{\max}$) labeled by Clifford vacua $|\Omega_{s_{\max}}\rangle$ as follows:

$$\begin{aligned}|\Omega_2\rangle &= |0\rangle\,,\\ |\Omega_3\rangle &= \left[\xi^A(1)\eta^K(1) - \xi^A(2)\eta^K(2)\right]|0\rangle\,,\\ |\Omega_4\rangle &= [\xi^{A_1}(1)\xi^{A_2}(1)\eta^{K_1}(1)\eta^{K_2}(1) - 4\xi^{[A_1}(1)\xi^{A_2]}(2)\eta^{[K_1}(1)\eta^{K_2]}(2) +\\ &\quad+\xi^{A_1}(2)\xi^{A_2}(2)\eta^{K_1}(2)\eta^{K_2}(2)]|0\rangle\,,\\ &\ \ \vdots\\ |\Omega_{n+2}\rangle &= \sum_{m=0}^{n}(-1)^m\binom{n}{m}^2\xi^{[A_1}(1)\cdots\xi^{A_m}(1)\xi^{A_{m+1}}(2)\cdots\xi^{A_n]}(2)\,\cdot\\ &\quad\cdot\eta^{[K_1}(1)\cdots\eta^{K_m}(1)\eta^{K_{m+1}}(2)\cdots\eta^{K_n]}(2)|0\rangle\,,\\ &\ \ \vdots\end{aligned}$$

where $[A_1 \ldots A_n]$ denotes graded symmetrization. The symmetric and anti-symmetric parts of the tensor product are given by

$$
\begin{aligned}
(\mathcal{D} \otimes \mathcal{D})_S &= \sum_{\ell=0,1,\ldots} \mathcal{D}_{s_{\max}=2\ell+2}\,, \\
(\mathcal{D} \otimes \mathcal{D})_A &= \sum_{\ell=0,1,\ldots} \mathcal{D}_{s_{\max}=2\ell+3}\,,
\end{aligned}
\tag{A.15}
$$

where ℓ is a level index. In the symmetric product, the level $\ell = 0$ multiplet is the $D = 5$, $N = 8$ supergravity multiplet; the level $\ell \geq 1$ multiplets have spin range 4 and consist of $256 + 256$ states. The $\mathrm{SU}(2,2) \times \mathrm{SU}(4) \times \mathrm{U}(1)_Y$ content of these multiplets [6] is listed in table 3.

B. Harmonic analysis on AdS_5

To determine the $\mathrm{SO}(4,2)$ content of the spectrum, we shall follow the technique used in [16, 17] which is based on the analytic continuation of AdS_5 to the five-sphere, and consequently the group $\mathrm{SO}(4,2)$ to $\mathrm{SO}(6)$. The Casimir eigenvalues for an $\mathrm{SO}(4,2)$ representation $D(j_L, j_R; E_0)$ and an $\mathrm{SO}(6)$ representation with highest weight labels (n_1, n_2, n_3) $(n_1 \geq n_2 \geq |n_3|)$ are

$$
\begin{aligned}
C_2[\mathrm{SO}(4,2)] &= E_0(E_0 - 4) + 2j_L(j_L + 1) + 2j_R(j_R + 1) \\
C_2[\mathrm{SO}(6)] &= n_1(n_1 + 4) + n_2(n_2 + 2) + n_3^2\,,
\end{aligned}
\tag{B.1}
$$

where the continuation requires the identification:

$$
\nabla^2|_{AdS_5} \to -\nabla^2|_{S^5}\,, \qquad E_0 = -n_1\,, \quad n_2 = j_L + j_R\,, \quad n_3 = j_L - j_R\,. \tag{B.2}
$$

The $\mathrm{SO}(6)$ Casimir is related to the Laplacian acting on a tensor T on S^5 in an irrep R of $\mathrm{SO}(5)$, which we expand as

$$
T_{\alpha_1 \ldots \alpha_{2s}}(x) = \sum_{\substack{(n_1,n_2,n_3) \\ p}} T_p^{(n_1,n_2,n_3)} D^{(n_1,n_2,n_3)}_{\alpha_1 \ldots \alpha_{2s},p}(L_x^{-1})\,, \tag{B.3}
$$

by the following formula

$$
-\nabla^2|_{S^5} D^{(n_1,n_2,n_3)}_{\alpha_1 \ldots \alpha_{2s},p} = \left(C_2[\mathrm{SO}(6)] - C_2[\mathrm{SO}(5)]\right) D^{(n_1,n_2,n_3)}_{\alpha_1 \ldots \alpha_{2s},p}\,. \tag{B.4}
$$

In (B.3), L_x is a coset representative of a point $x \in S^5$ and (n_1, n_2, n_3) label all $\mathrm{SO}(6)$ representations containing R, namely those which satisfy the embedding $n_1 \geq m_1 \geq n_2 \geq m_2 \geq |n_3|$ where (m_1, m_2) $(m_1 \geq m_2 \geq 0)$ are the highest weight labels of R. The $\mathrm{SO}(5)$ Casimir is given by

$$
C_2[\mathrm{SO}(5)] = m_1(m_1 + 3) + m_2(m_2 + 1)\,. \tag{B.5}
$$

Using the notation introduced in (5.1), the SO(5) highest weight labels of $\Phi^{(r;0)}_{\alpha_1\ldots\alpha_{2s}}$ $(0 \leq r \leq s)$ are

$$m_1 = s\,, \qquad m_2 = s - r\,. \tag{B.6}$$

The dimension of this representation is given by

$$d_{m_1,m_2} = \frac{2}{3}\left(m_1 + \frac{3}{2}\right)\left(m_2 + \frac{1}{2}\right)(m_1 + m_2 + 2)(m_1 - m_2 + 1)\,. \tag{B.7}$$

In case of integer spin s, the irrep R corresponds to an SO(4,1) Young tableaux with m_1 boxes in the first row and m_2 boxes in the second row.

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JHEP09(2001)025